

A Level  $\cdot$  AQA  $\cdot$  Maths



**Exam Questions** 

## **Partial Fractions**

Partial Fractions with Linear Denominators / Partial Fractions with Squared Linear Denominators

Total Marks	/195
Very Hard (9 questions)	/55
Hard (9 questions)	/53
Medium (9 questions)	/51
Easy (8 questions)	/36

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# **Easy Questions**

1 Express

$$\frac{1}{x(x+1)}$$

as partial fractions.

(3 marks)

- **2** (i) Factorise  $x^2 + 5x 6$ .
  - (ii) Hence, or otherwise, express

$$\frac{(5x+16)}{x^2+5x-6}$$

as partial fractions.

(4 marks)

3 Express

$$\frac{6x-13}{x^2-5x+6}$$

as partial fractions.

4 Write

$$\frac{2(x-11)}{x^2+2x-15}$$

in the form

$$\frac{A}{x+5} + \frac{B}{x-3}$$

where A and B are integers to be found.

(3 marks)

**5** Show that

$$\frac{3x+8}{(x+1)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

where  $\boldsymbol{A}$  and  $\boldsymbol{B}$  are integers to be found.

(3 marks)

**6** Express

$$\frac{5x^2 + 10x + 8}{x(x+2)^2}$$

in partial fractions.

(5 marks)

**7** Write

$$\frac{3x^2 - 6x + 2}{x^3 - 3x^2 + 2x}$$

in the form

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

where A, B and C are integers to be found.

(5 marks)

8 (a) (i) Briefly explain why

$$\frac{x^2 + x - 4}{x^2 + 4x + 3}$$

is an improper (algebraic) fraction.

(ii) Show that

$$(x^2 + x - 4) \div (x^2 + 4x + 3) = 1 - \frac{(3x + 7)}{x^2 + 4x + 3}$$

(4 marks)

**(b)** Express

$$\frac{3x+7}{x^2+4x+3}$$

as partial fractions.

(3 marks)

(c) Hence, show that

$$\frac{x^2 + x - 4}{x^2 + 4x + 3} = 1 - \frac{2}{x + 1} - \frac{1}{x + 3}$$

#### **Medium Questions**

1 Express  $\frac{2}{x(x+2)}$  as partial fractions.

(3 marks)

2 Write 
$$\frac{x-10}{(x+2)(x-1)}$$
 in the form  $\frac{A}{x+2} + \frac{B}{x-1}$ , where  $A$  and  $B$  are integers to be found.

(3 marks)

3 (a) It is given that

$$f(x) = x^3 - 13x + 12.$$

- (i) Show that f(1) = 0.
- (ii) Hence write down a factor of f(x).

(3 marks)

- **(b)** (i) Show that f(3) = 0.
  - (ii) Hence write down another factor of f(x).

(3 marks)

(c) Fully factorise f(x).

(2 marks)

(d) Express  $\frac{3x^2 - 13}{x^3 - 13x + 12}$  as partial fractions.

(4 marks)

4 Express  $\frac{7x+34}{x^2+9x+14}$  as partial fractions.

(4 marks)

**5 (a)** Use algebraic division to work out  $(x^2 + 4x - 6) \div (x^2 - x - 6)$ .

**(b)** Write 
$$\frac{5x}{(x+2)(x-3)}$$
 in the form  $\frac{A}{x+2} + \frac{B}{x-3}$ 

(3 marks)

(c) Hence, or otherwise, show that 
$$\frac{x^2 + 4x - 6}{x^2 - x - 6} = 1 + \frac{2}{x + 2} + \frac{3}{x - 3}$$

(3 marks)

**6** Express 
$$\frac{2x}{(x+1)(x-2)}$$
 as partial fractions.

(4 marks)

**7 (a)** Use the factor theorem to show that 
$$(x + 1)$$
 is a factor of  $x^3 - 3x^2 + 4$ .

(2 marks)

**(b)** Hence, or otherwise, fully factorise 
$$x^3 - 3x^2 + 4$$
.

(c) Show that 
$$\frac{19-8x}{x^3-3x^2+4}$$
 can be written in the form  $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$ , where  $A,B$  and  $C$  are integers to be found.

8 Express 
$$\frac{3x+11}{x^2+6x+9}$$
 as partial fractions.

(4 marks)

**9** Express 
$$\frac{x^2-5}{x^3-x^2-17x-15}$$
 as partial fractions.

(5 marks)

#### **Hard Questions**

1 Express 
$$\frac{12}{x^2+4x-5}$$
 as partial fractions.

(3 marks)

2 Write 
$$\frac{12x-6}{(x-3)(x+2)(x-1)}$$
 in the form  $\frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{x-1}$ , where  $A,B$  and  $C$  are integers to be found.

(4 marks)

3 (a) It is given that

$$f(x) = x^3 + 3x^2 - 4x - 12.$$

- (i) Show that f(-3) = 0
- (ii) Hence write down a factor of f(x).

(3 marks)

**(b)** Fully factorise f(x).

(c) Express 
$$\frac{3x^2 - 8x - 36}{x^3 + 3x^2 - 4x - 12}$$
 as partial fractions.

(3 marks)

4 Express 
$$\frac{2x^2 + 16x - 24}{x^3 - 2x^2 - 8x}$$
 as partial fractions.

(4 marks)

**5 (a)** Show that 
$$\frac{x^2+x+5}{x^2+3x+2}$$
 can be written in the form  $A-\frac{2x-3}{(x+1)(x+2)}$  , where  $A$  is an integer to be found.

**(b)** Express 
$$\frac{2x-3}{(x+1)(x+2)}$$
 in the form  $\frac{B}{x+1} + \frac{C}{x+2}$  where  $B$  and  $C$  are integers to be found.

(3 marks)

(c) Using your values for 
$$A$$
,  $B$  and  $C$  write  $\frac{x^2+x+5}{x^2+3x+2}$  in the form  $A+\frac{B}{x+1}+\frac{C}{x+2}$ 

(3 marks)

**6** Express 
$$\frac{2x}{x^2 - 9x - 52}$$
 as partial fractions.

(5 marks)

**7 (a)** Use the factor theorem to show that 
$$(x-2)$$
 is a factor of  $x^3 + 4x^2 - 3x - 18$ .

(2 marks)

**(b)** Hence, or otherwise, fully factorise 
$$x^3 + 4x^2 - 3x - 18$$
.

(3 marks)

(c) Show that 
$$\frac{7x^2 + 32x + 8}{x^3 + 4x^2 - 3x - 18}$$
 can be written in the form  $\frac{A}{x - 2} + \frac{B}{x + 3} + \frac{C}{(x + 3)^2}$ , where  $A, B$  and  $C$  are integers to be found.

8 Express 
$$\frac{3x^2 - 22x + 25}{x^3 - 7x^2 + 8x + 16}$$
 as partial fractions.

(5 marks)

**9** Express 
$$\frac{2x^2-3}{x^3+4x^2-3x-18}$$
 as partial fractions.

(5 marks)

### **Very Hard Questions**

1 Express 
$$\frac{8}{x^3 - 6x^2 + 8x}$$
 as partial fractions.

(5 marks)

2 Write 
$$\frac{13x^2-10}{x^3-x^2-2x}$$
 in the form  $\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-2}$ , where  $A,B$  and  $C$  are integers to be found.

(5 marks)

3 Show that 
$$\frac{34-3x^2-x}{x^3-5x^2+3x+9}$$
 can be written in the form  $\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$ , where  $A$ ,  $B$  and  $C$  are integers to be found.

(5 marks)

4 Express 
$$\frac{x^2-3}{(x-3)(x+2)^2}$$
 as partial fractions.

(4 marks)

5 Write 
$$\frac{x^2+3x+10}{x^2+8x+15}$$
 in the form  $A+\frac{B}{x+3}+\frac{C}{x+5}$ , where  $A,B$  and  $C$  are integers to be found.

(4 marks)

6 (a) It is given that

$$f(x) = x^3 + 3x^2 - 4x - 12.$$

- (i) Show that f(-3) = 0.
- (ii) Hence, fully factorise f(x).

(6 marks)

**(b)** Express 
$$\frac{3x^2 - 8x - 36}{f(x)}$$
 as partial fractions.

(4 marks)

7 Express 
$$\frac{2x-5}{x^3-3x^2-13x+15}$$
 as partial fractions.

(10 marks)

8 Express 
$$\frac{2(3x^3 + x^2 - 17x + 4)}{x(x-1)(x-4)(x+2)}$$
 as partial fractions.

(5 marks)

9 Given that 
$$x^4 - 4x^3 - 2x^2 + 12x + 9 = (x^2 - 6x + 9)(x^2 + 2x + 1)$$
, express 
$$\frac{5x^3 - 15x^2 + 19x + 7}{x^4 - 4x^3 - 2x^2 + 12x + 9}$$
 as partial fractions.

(7 marks)