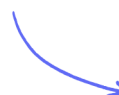


Estimating Gradients & Areas under Graphs

Finding Gradients of Tangents / Finding Areas under Graphs

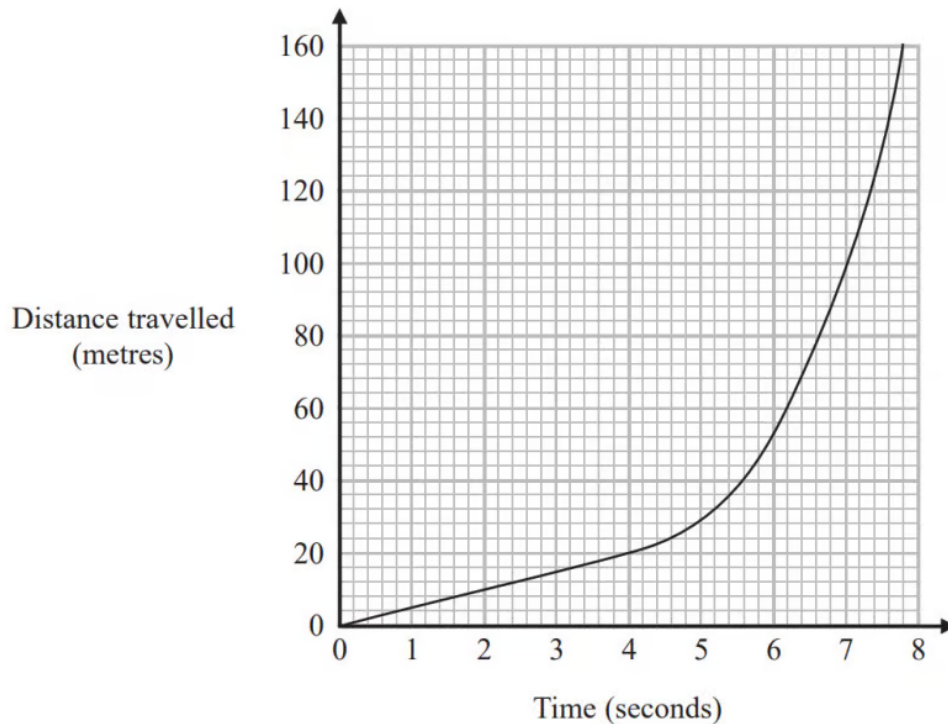
Medium (16 questions)	/56
Hard (12 questions)	/51
Very Hard (6 questions)	/34
Total Marks	/141

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Medium Questions

- 1 The distance-time graph shows information about part of a car journey.

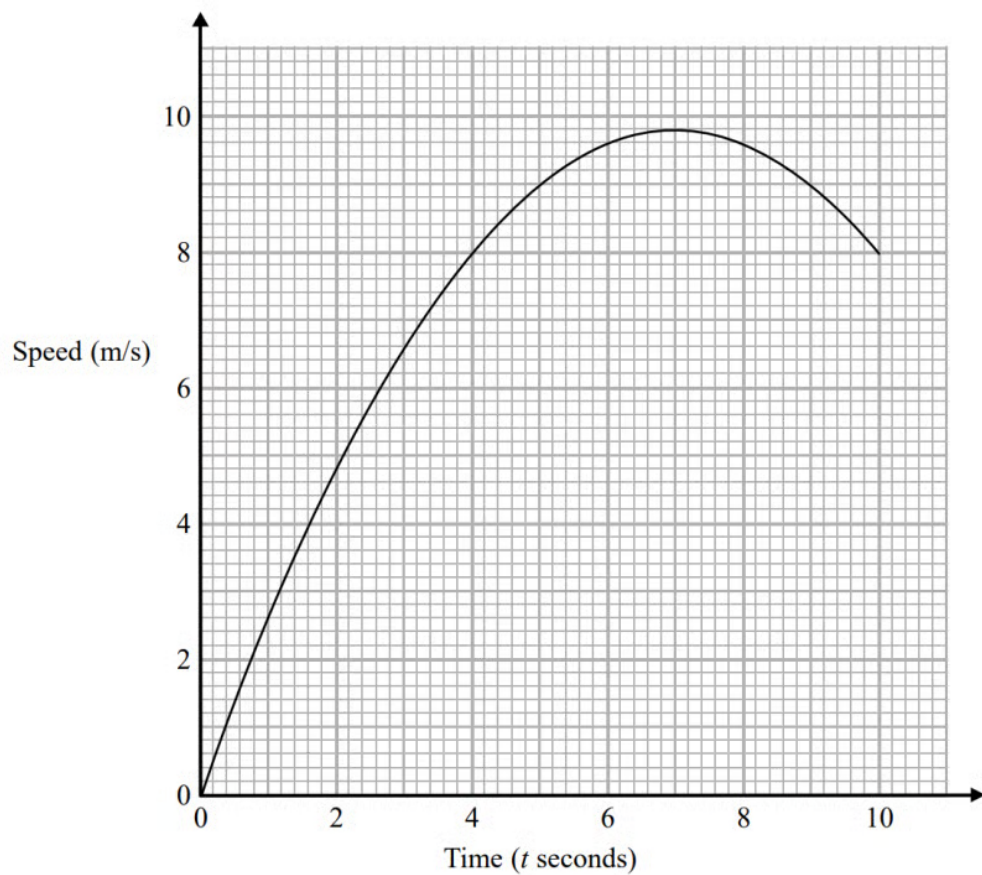


Use the graph to estimate the speed of the car at time 5 seconds.

(3 marks)

- 2 (a) Karol runs in a race.

The graph shows her speed, in metres per second, t seconds after the start of the race.



Calculate an estimate for the gradient of the graph when $t = 4$. You must show how you get your answer.

(3 marks)

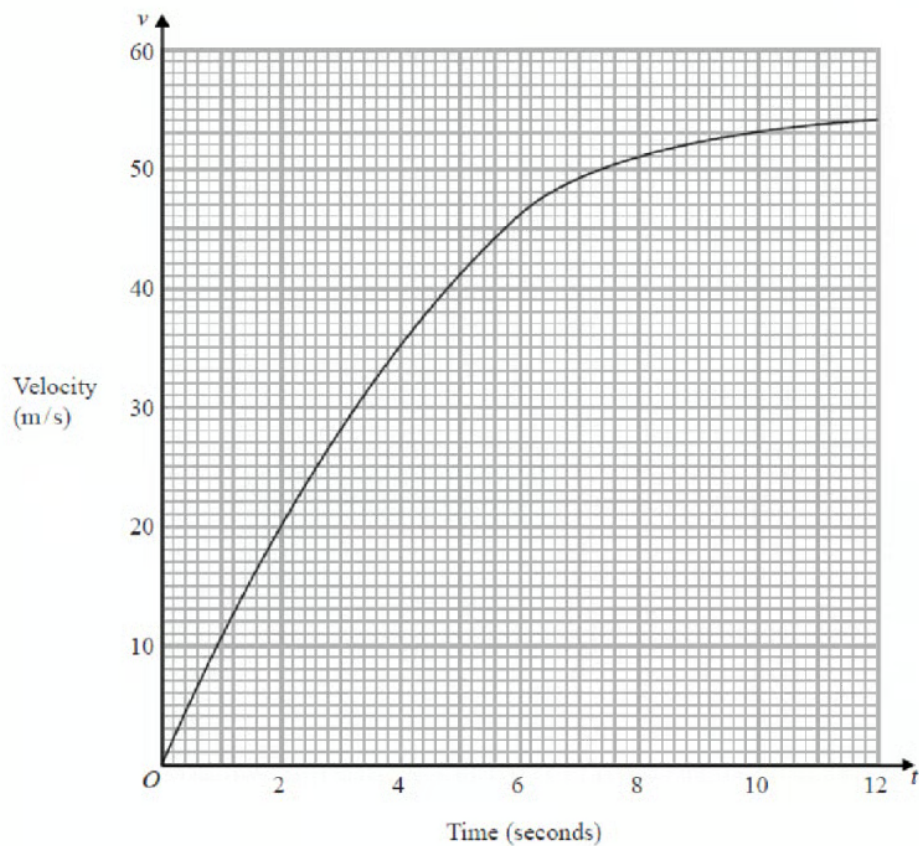
(b) Describe fully what your answer to part (a) represents.

(2 marks)

(c) Explain why your answer to part (a) is only an estimate.

(1 mark)

- 3 (a)** The graph shows information about the velocity of a parachutist after jumping from a plane.



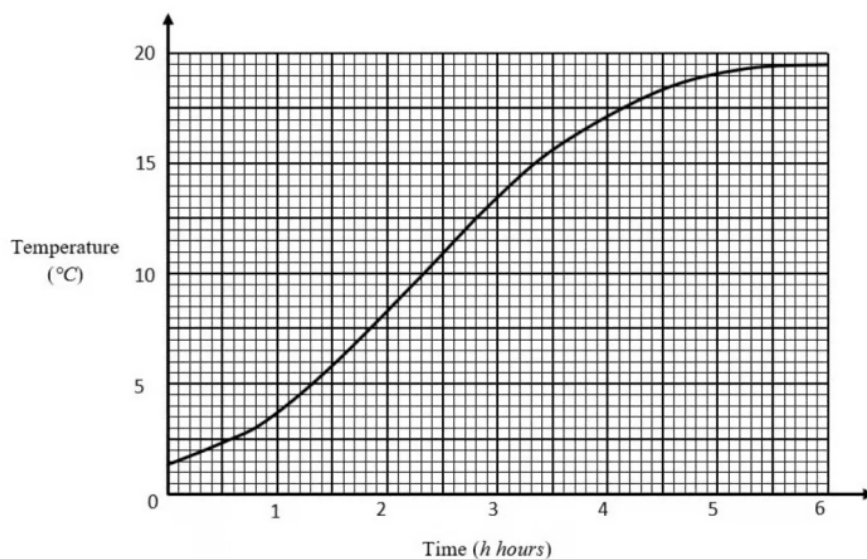
By drawing a suitable tangent, find an estimate of the gradient of the curve after 3 seconds.

(2 marks)

- (b)** Interpret the value of the gradient.

(1 mark)

- 4 (a)** The graph shows the temperature of a fish tank over the first 6 hours after a heater is added.



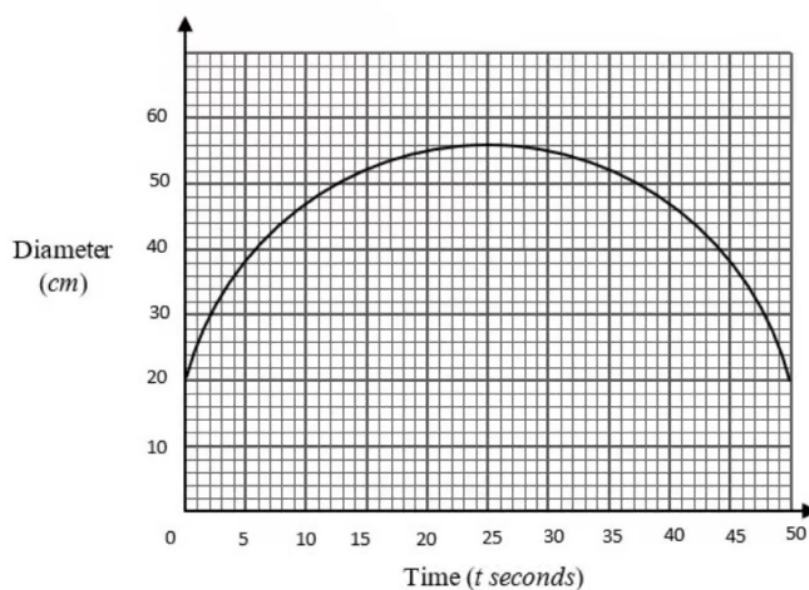
By drawing a suitable tangent, find an estimate of the gradient of the curve when $h = 3$.
(2 marks)

- (b)** Interpret the value of the gradient.

(1 mark)

5 (a) A fish bowl is being filled with water.

The graph shows how the diameter of the surface of the water changes with time.



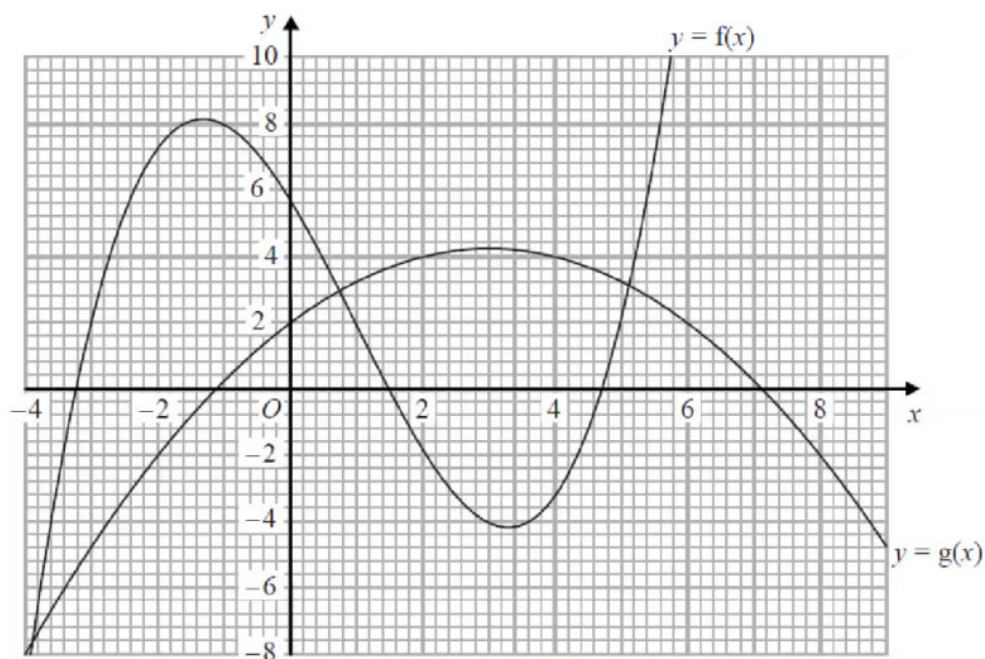
Find an estimate for the gradient at $t = 10$.

(2 marks)

(b) Give an interpretation of the gradient.

(1 mark)

6 (a) The diagram shows parts of the graphs of $y=f(x)$ and $y=g(x)$



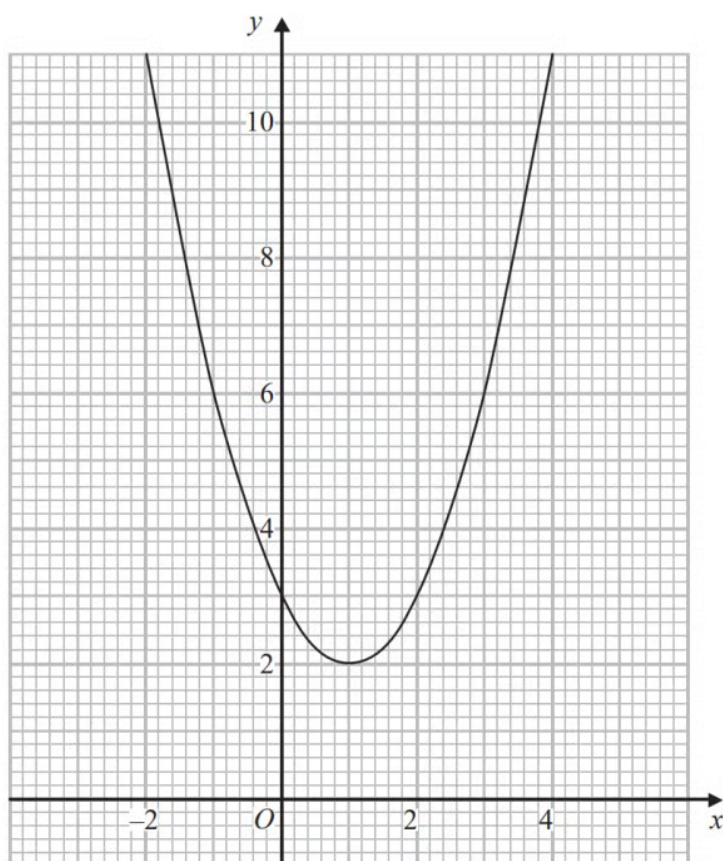
Write down the value of x where the gradient of the curve $y=g(x)$ is zero.

(1 mark)

(b) Calculate an estimate for the gradient of the curve $y = f(x)$ at the point on the curve where $x = 4$.

(2 marks)

7 (a) The diagram shows part of the graph of $y = x^2 - 2x + 3$



By drawing a suitable straight line, use your graph to find estimates for the solutions of $x^2 - 3x - 1 = 0$

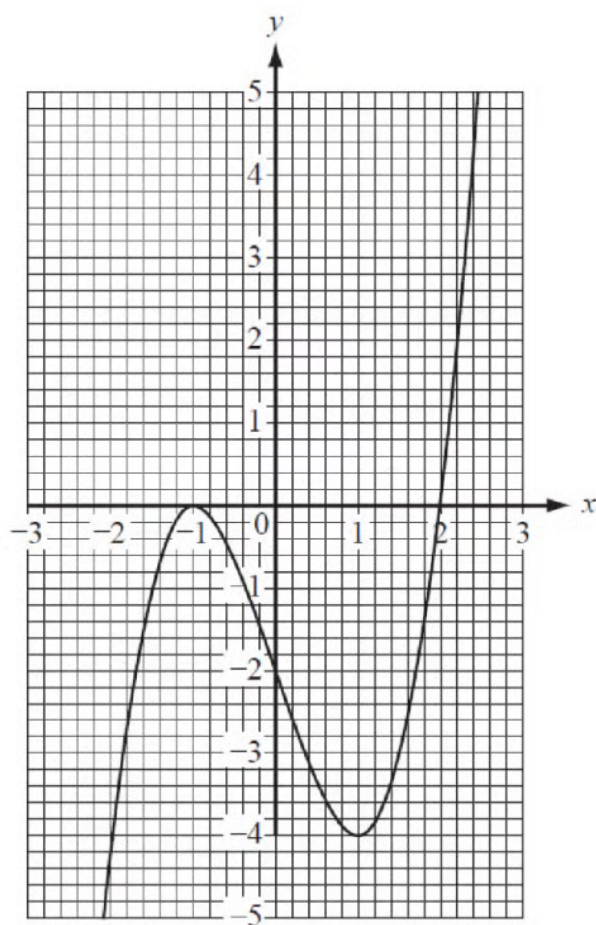
(2 marks)

(b) P is the point on the graph of $y = x^2 - 2x + 3$ where $x = 2$

Calculate an estimate for the gradient of the graph at the point P .

(3 marks)

8 (a) The curve $y = x^3 - 3x - 2$ is shown on the grid.



Write down the co-ordinates of the points where the gradient of the curve is zero.

(2 marks)

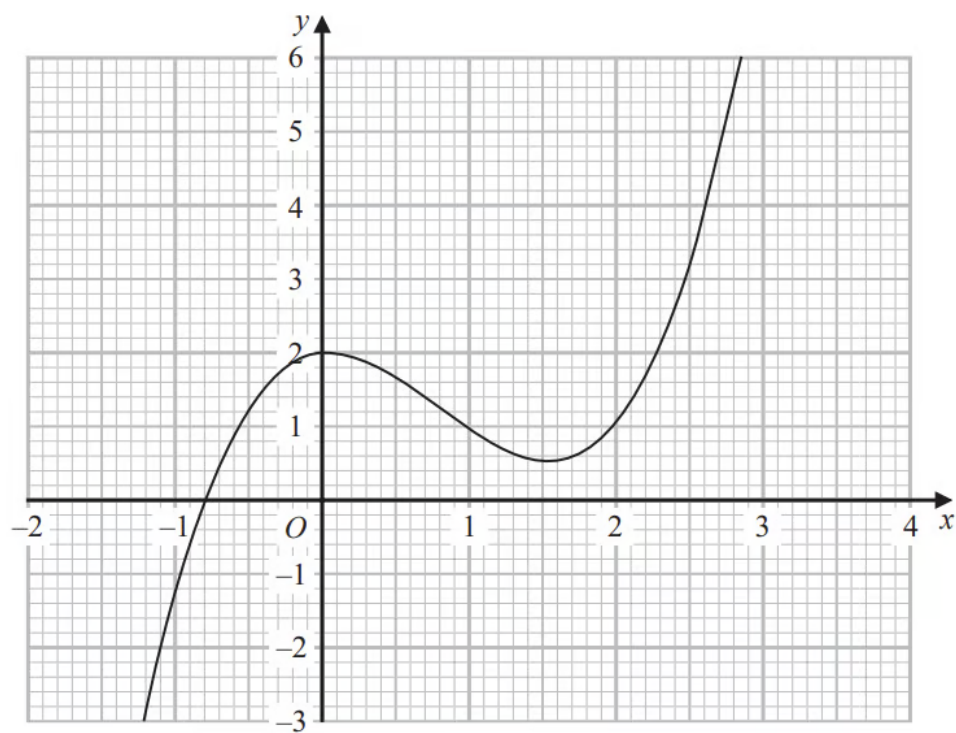
(b) Write down the range of values of x when the gradient of the curve is negative.

(1 mark)

(c) Find an estimate of the gradient of the curve when $x = 2$.

(2 marks)

9 Part of the curve with equation $y = f(x)$ is shown on the grid.



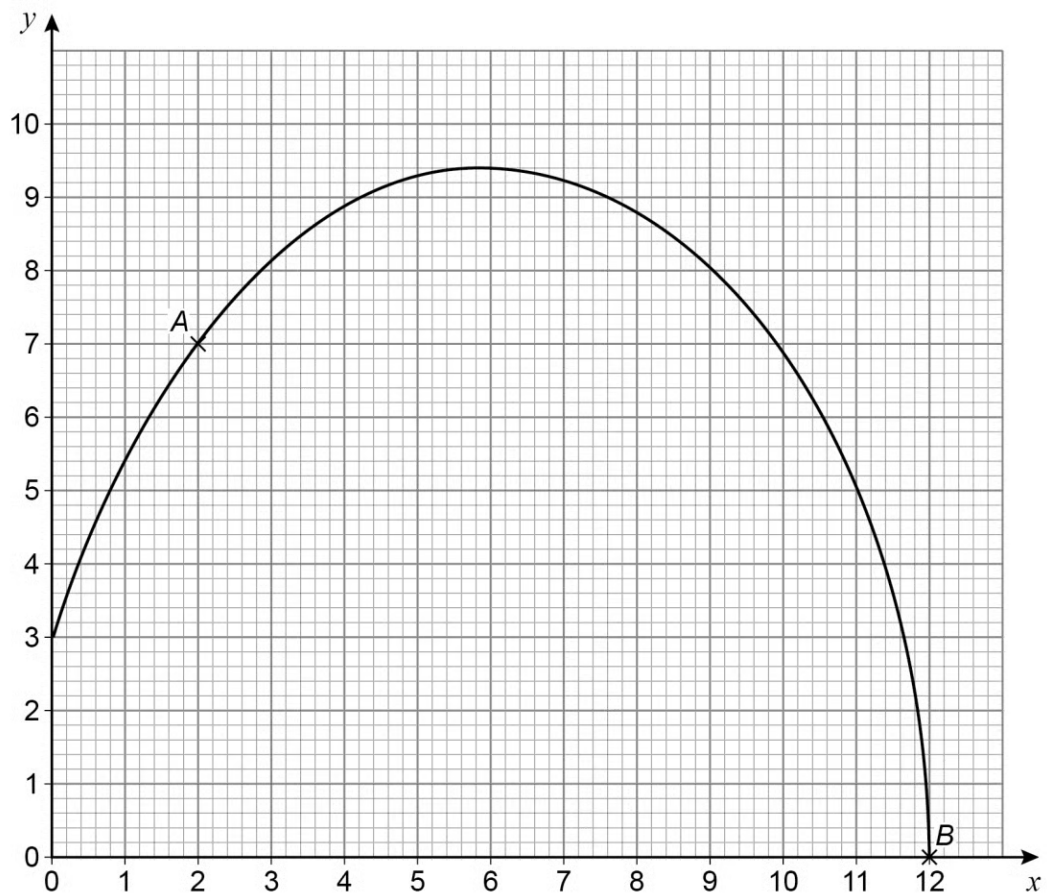
Find an estimate for the gradient of the curve at the point where $x = 2$

Show your working clearly.

(3 marks)

10 (a) A and B are points on a curve.

A is $(2, 7)$ B is $(12, 0)$



Work out the instantaneous rate of change of y with respect to x at point A .

(2 marks)

- (b) The average rate of change of y with respect to x between points A and B is worked out.

Which statement is correct?

Tick **one** box.

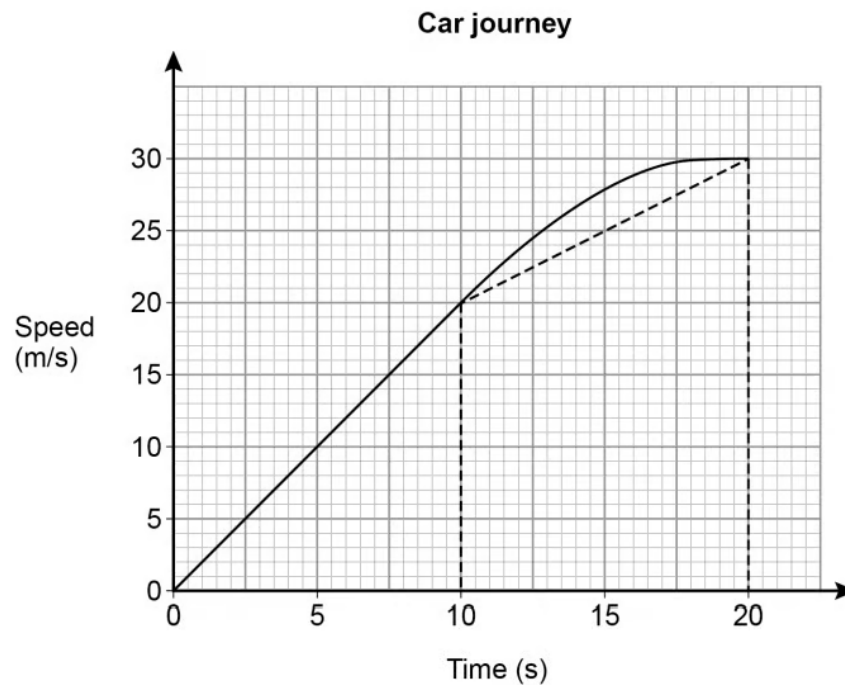
☐	It is positive.
☐	It is zero.
☐	It is negative.
☐	You cannot tell if it is positive or negative.

(1 mark)

11 (a) The speed-time graph shows 20 seconds of a car journey.

Harry wants to estimate the distance the car travels in this time.

He uses a triangle and a trapezium, as shown, to estimate the area under the graph.



Complete Harry's method to estimate the distance the car travels.

.....m

(3 marks)

(b) For this journey, which of these is true for Harry's method?

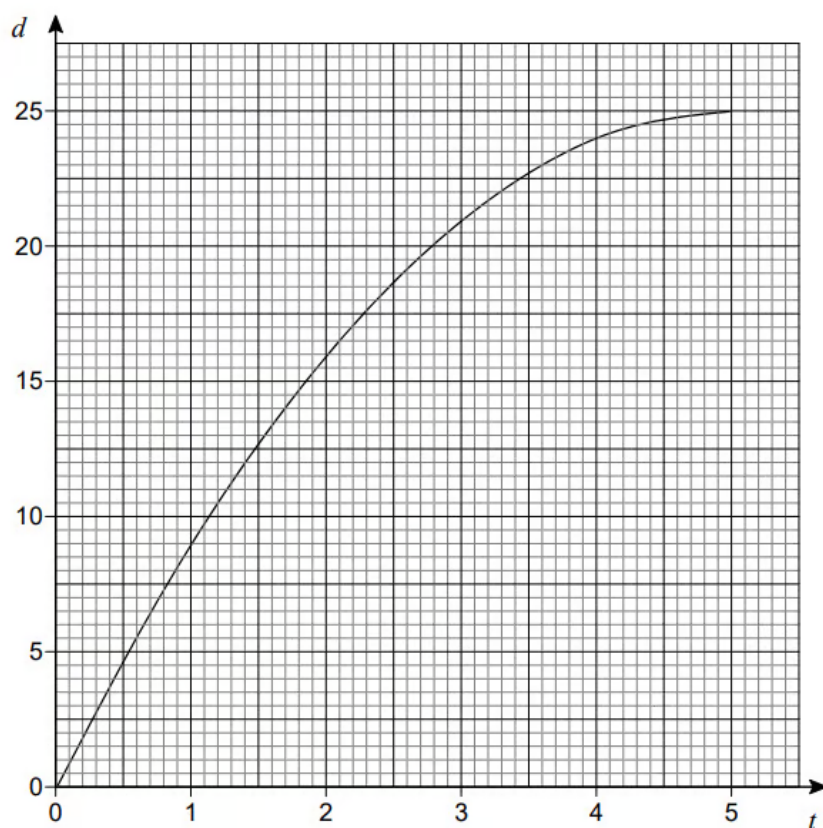
Tick **one** box.

- ☐ It works out an overestimate of the distance
- ☐ It works out an underestimate of the distance

☐ It could work out an overestimate or an underestimate of the distance

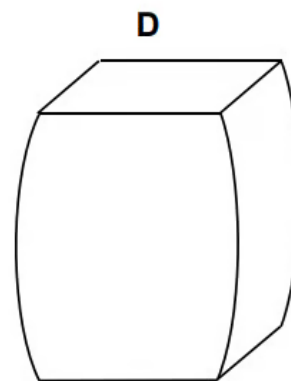
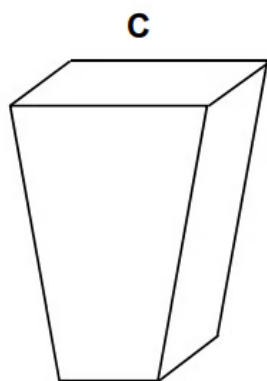
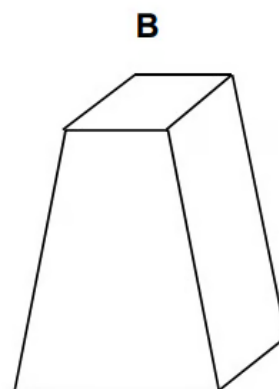
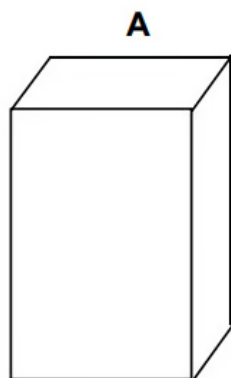
(1 mark)

- 12 (a)** A container is filled with water in 5 seconds.
The graph shows the depth of water, d cm, at time t seconds.



The water flows into the container at a constant rate.

Which diagram represents the container?
Circle the correct letter.



(1 mark)

(b) Use the graph to estimate the rate at which the depth of water is increasing at 3 seconds.

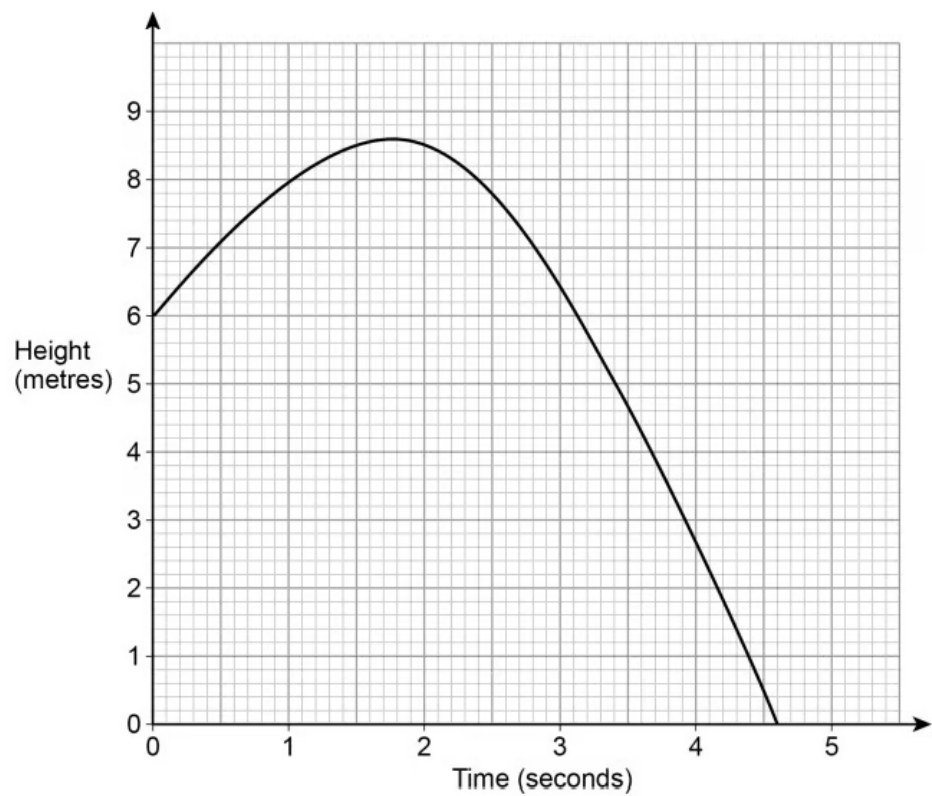
You **must** show your working.

.....cm/s

(2 marks)

13 A ball is thrown from a point 6 metres above the ground.

The graph shows the height of the ball above the ground, in metres.



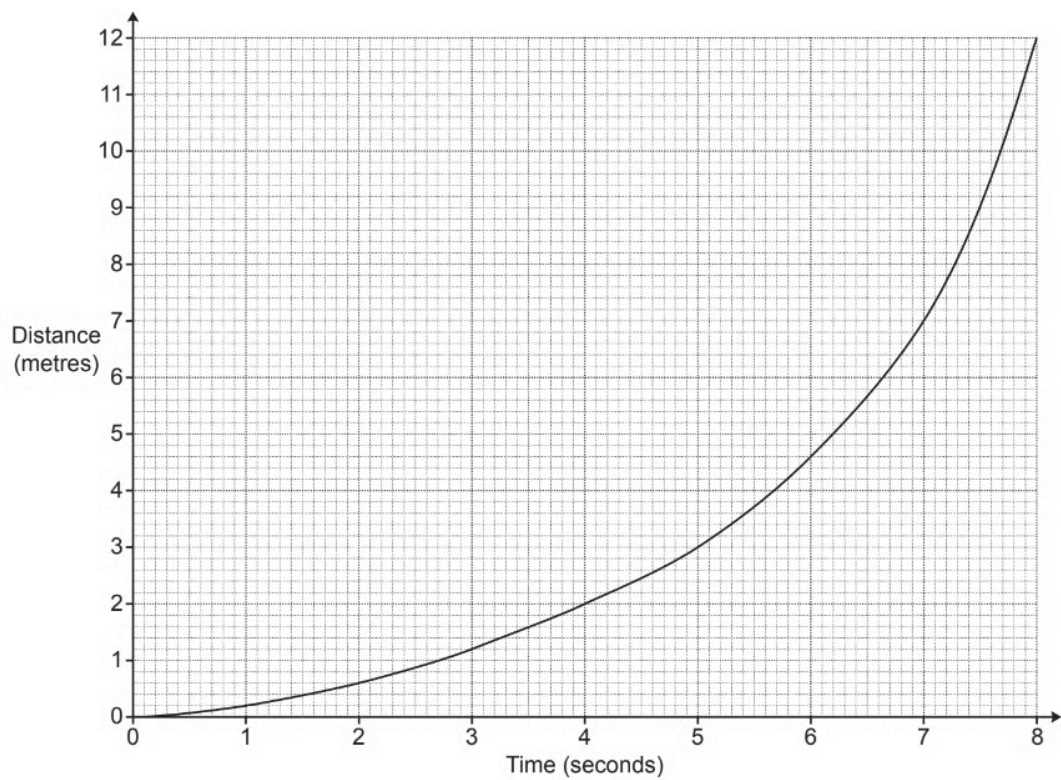
Estimate the speed of the ball, in m/s, after 1 second.

You **must** show your working.

.....m/s

(2 marks)

14 The graph shows the distance travelled by a particle over 8 seconds.

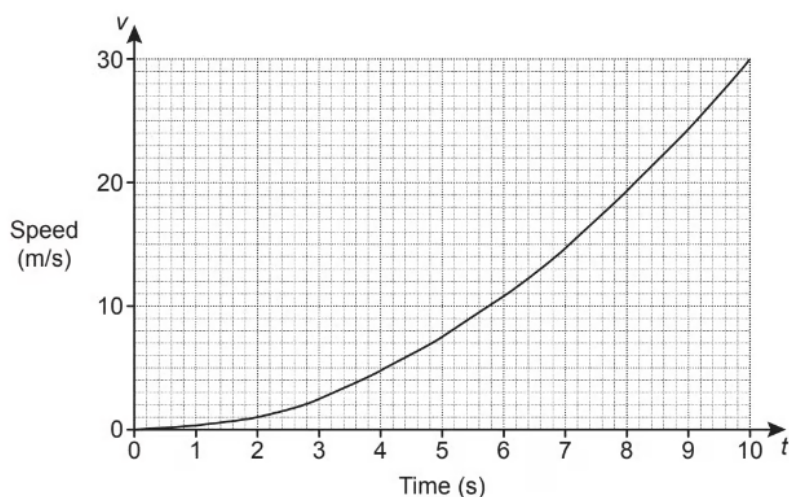


Estimate the speed of the particle at 5 seconds.

..... m/s

(4 marks)

- 15 The graph shows the speed, v metres per second (m/s), of a car at time t seconds.

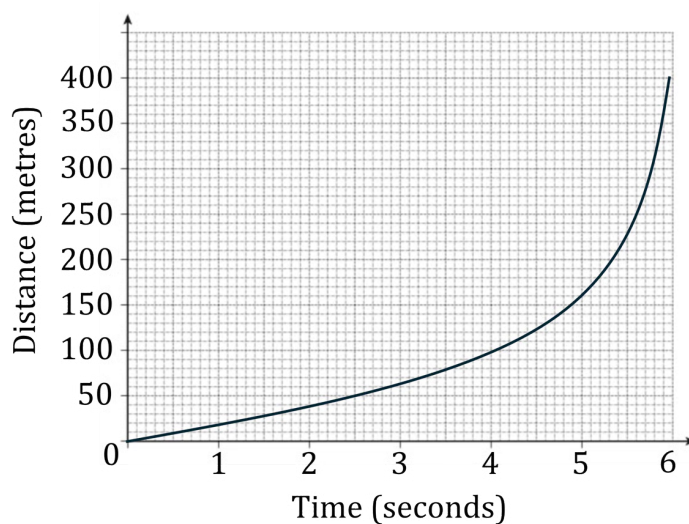


Use the graph to estimate the acceleration at $t = 7$.

.....m/s²
(3 marks)

- 16 An object falls from rest.

Here is the distance-time graph for the distance (d metres) fallen by the object t seconds after it starts to fall.



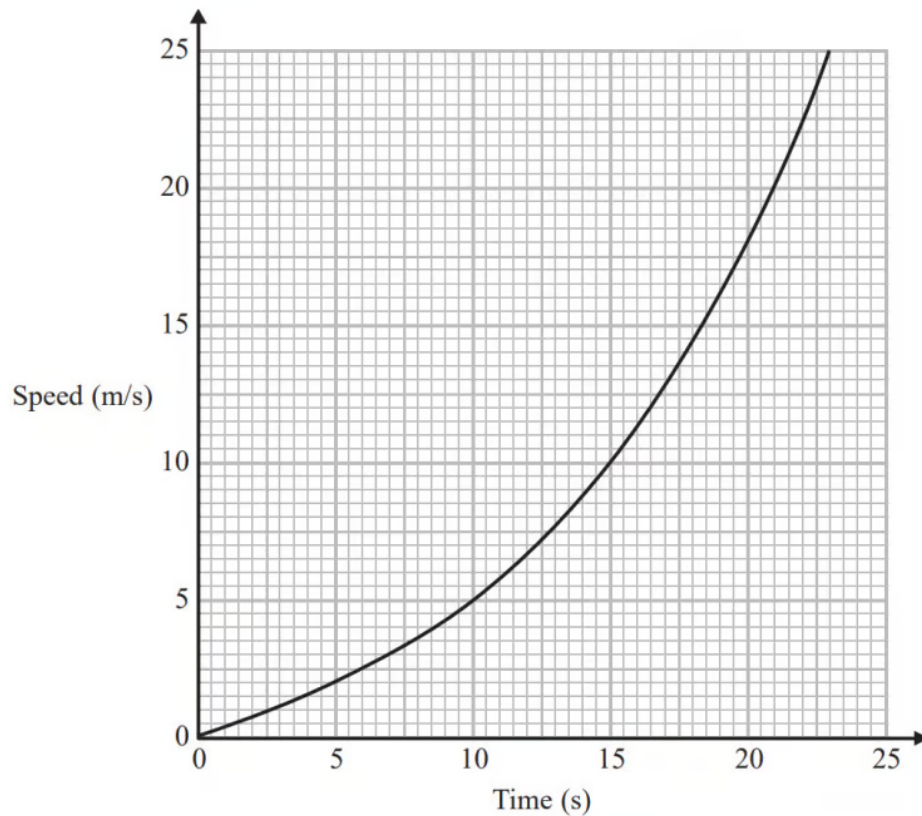
Work out an estimate for the gradient of the graph at $t = 5$.

You must show how you get your answer.

(3 marks)

Hard Questions

1 (a) Here is a speed-time graph for a train.



Work out an estimate for the distance the train travelled in the first 20 seconds.
Use 4 strips of equal width.

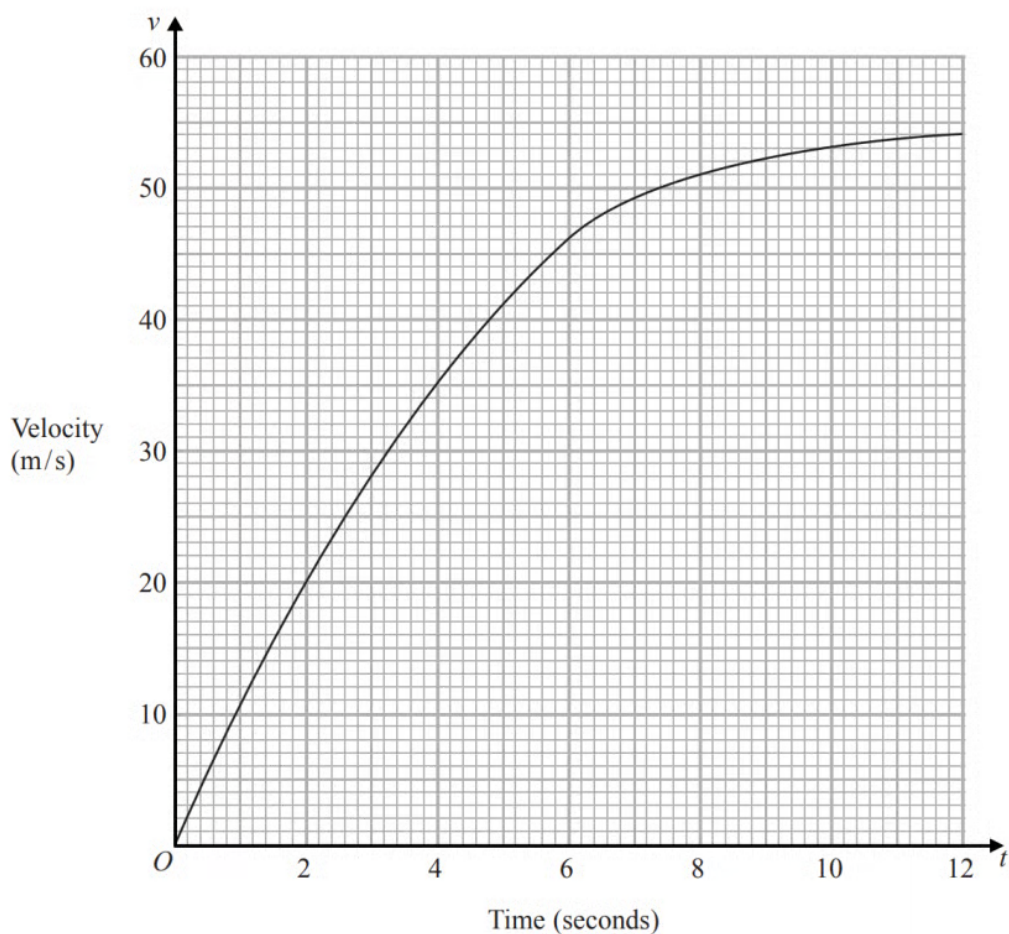
(3 marks)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the train travelled?

Give a reason for your answer.

(1 mark)

- 2 (a)** The graph shows information about the velocity, v m/s, of a parachutist t seconds after leaving a plane.



Work out an estimate for the acceleration of the parachutist at $t = 6$

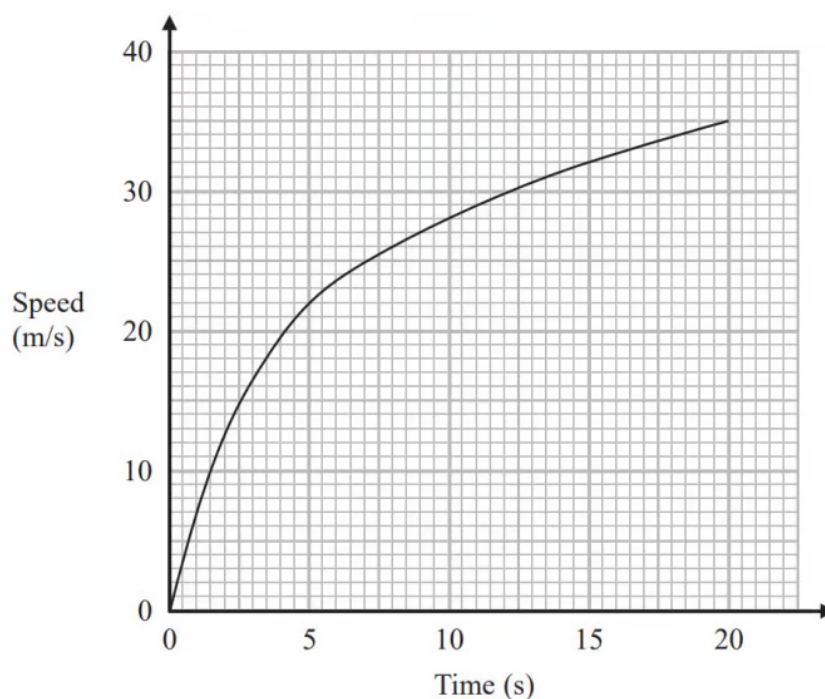
(2 marks)

- (b)** Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane.

Use 3 strips of equal width.

(3 marks)

- 3 (a)** The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.



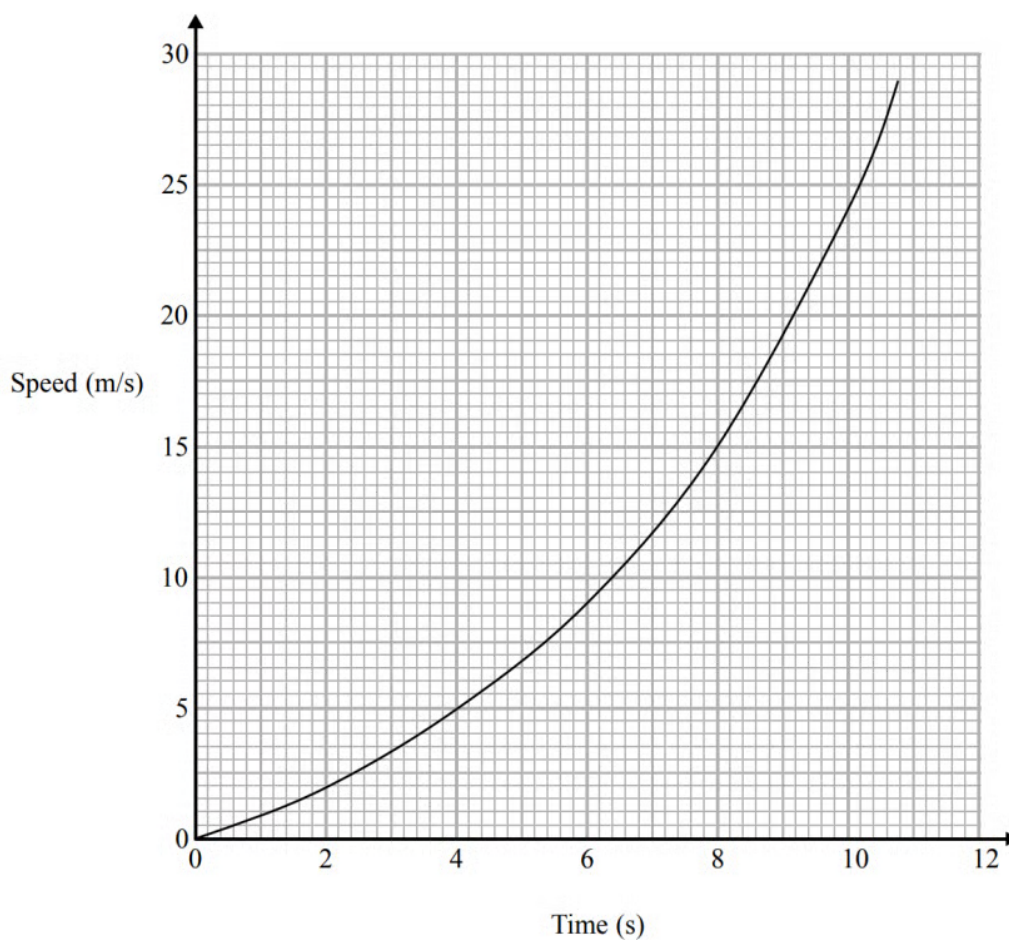
Work out an estimate for the distance the car travelled in the first 20 seconds.
Use 4 strips of equal width.

(3 marks)

- (b)** Is your answer to part (a) an underestimate or an overestimate of the actual distance the car travelled in the first 20 seconds? Give a reason for your answer.

(1 mark)

4 (a) Here is a speed-time graph for a car.



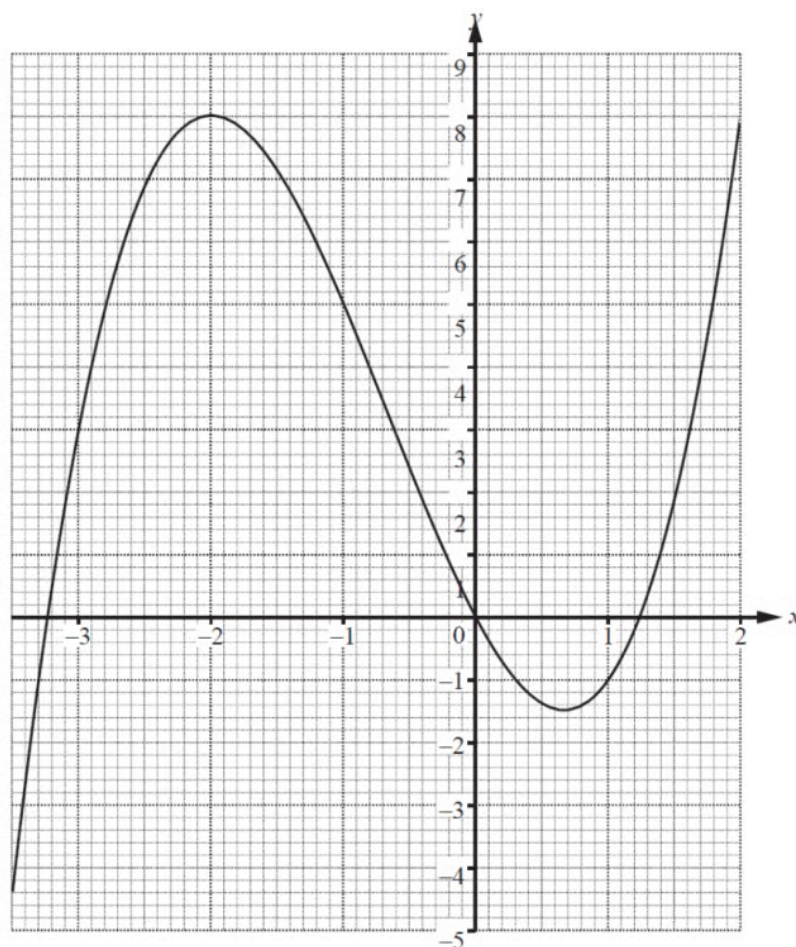
Work out an estimate for the distance the car travelled in the first 10 seconds. Use 5 strips of equal width.

(3 marks)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance? Give a reason for your answer.

(1 mark)

5 (a) The curve $y = x^3 + 2x^2 - 4x$ is shown on the grid.



By drawing a suitable tangent, find an estimate of the gradient of the curve when $x = 1$.

(2 marks)

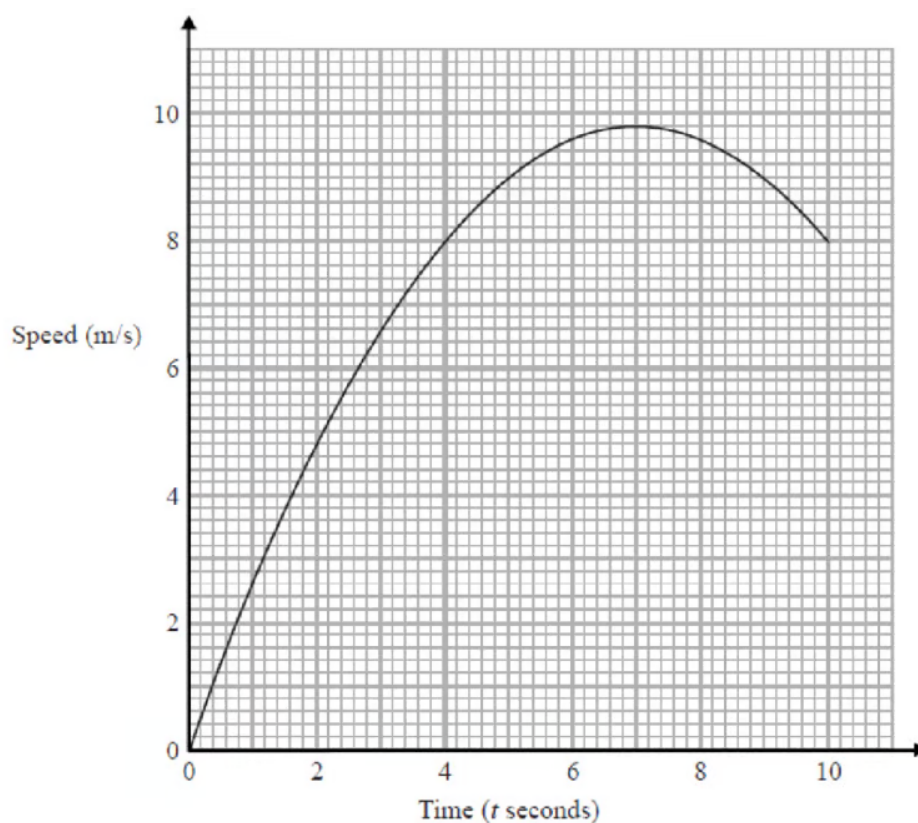
- (b) A point D lies on the curve.
The x co-ordinate of D is negative.
The gradient of the tangent at D is 0.

Write down the co-ordinates of D .

(1 mark)

6 (a) Ellie runs a race.

The graph shows Ellie's speed in the first 10 seconds after the start of the race.



By drawing a suitable tangent, work out the acceleration when $t = 9$.
Give the units of your answer.

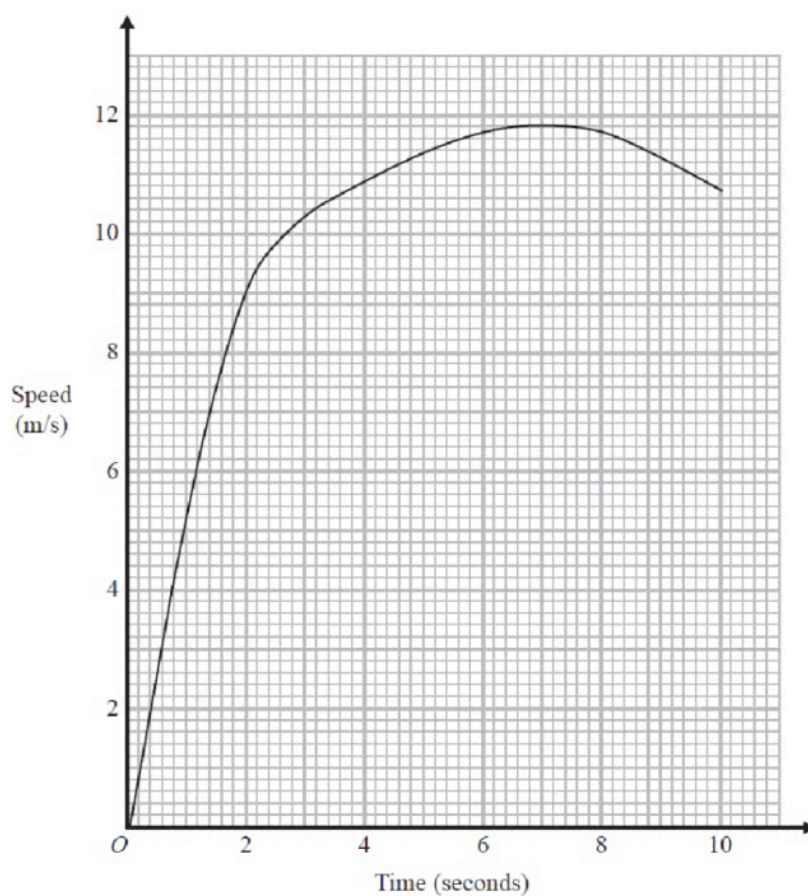
(3 marks)

(b) Describe what happens to Ellie after 7 seconds.

(1 mark)

7 (a) Olympic medallist Usain runs in a race.

The graph shows his speed, in metres per second (m/s), during the first 10 seconds of the race.



Use the graph to find how long it took Usain to reach his top speed.

(1 mark)

(b) Work out an estimate for Usain's acceleration at 2 seconds.

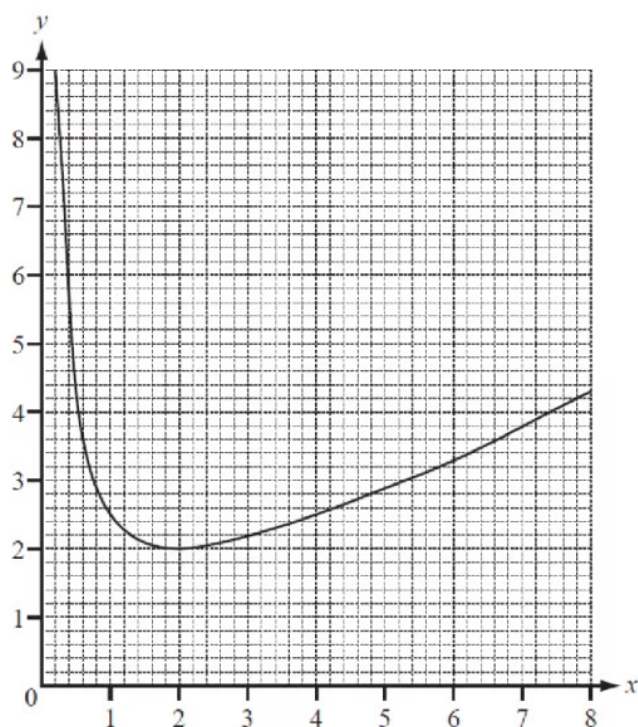
Give the units of your answer.

(3 marks)

(c) Calculate the difference in Usain's acceleration between 2 and 6 seconds.

(3 marks)

8 (a) The diagram shows the graph of $y = \frac{x}{2} + \frac{2}{x}$ for $0 < x \leq 8$.



Use the graph to solve the equation $\frac{x}{2} + \frac{2}{x} = 3$.

(2 marks)

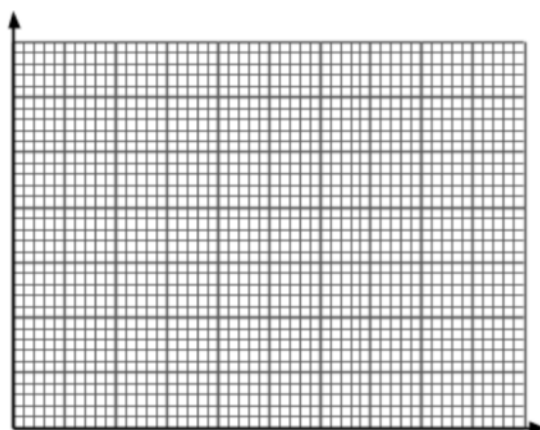
(b) By drawing a suitable tangent, find an estimate of the gradient of the graph when $x = 1$.

(3 marks)

- 9 (a)** Clare emptied a tank and recorded the depth of water each minute.

Time (t minutes)	0	1	2	3	4	5	6	7	8	9	10
Depth (m metres)	30	29.5	29	28	27	26	24.5	22.5	19.5	15	9

Plot the graph of depth against time.



(2 marks)

- (b)** Work out the average rate of decrease of the depth of the water in Clare's tank between $t = 0$ and $t = 10$.

(2 marks)

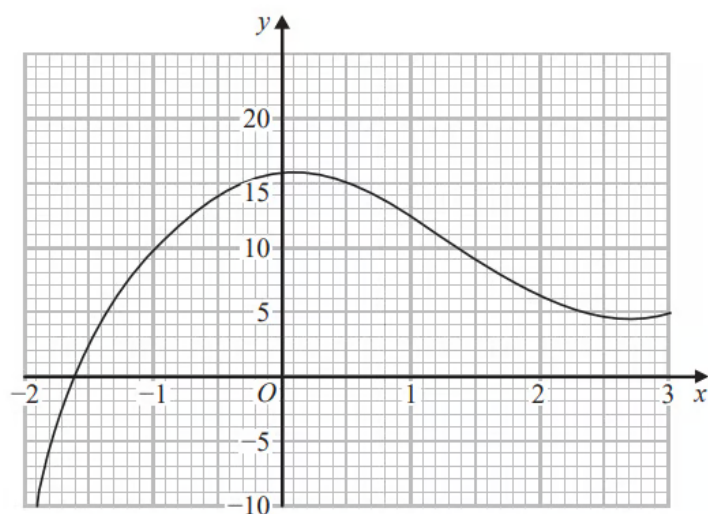
- (c)** Find an estimate for the gradient at $t = 7$.

(2 marks)

- (d)** Give an interpretation of part (c).

(1 mark)

10 Part of the curve with equation $y = h(x)$ is shown on the grid.

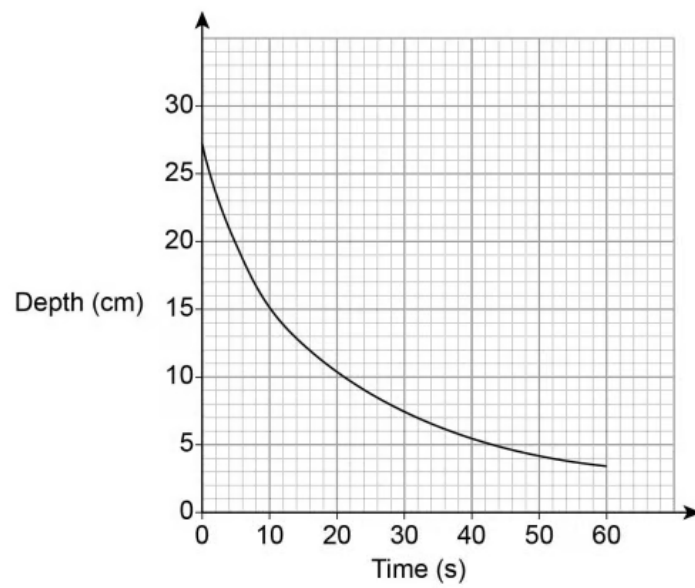


Find an estimate for the gradient of the curve at the point where $x = -0.5$
Show your working clearly.

(3 marks)

11 Liquid is leaking out of a container.

The graph shows the depth of the liquid for 60 seconds.



Use the graph to work out an estimate of the rate of decrease of depth at 10 seconds.

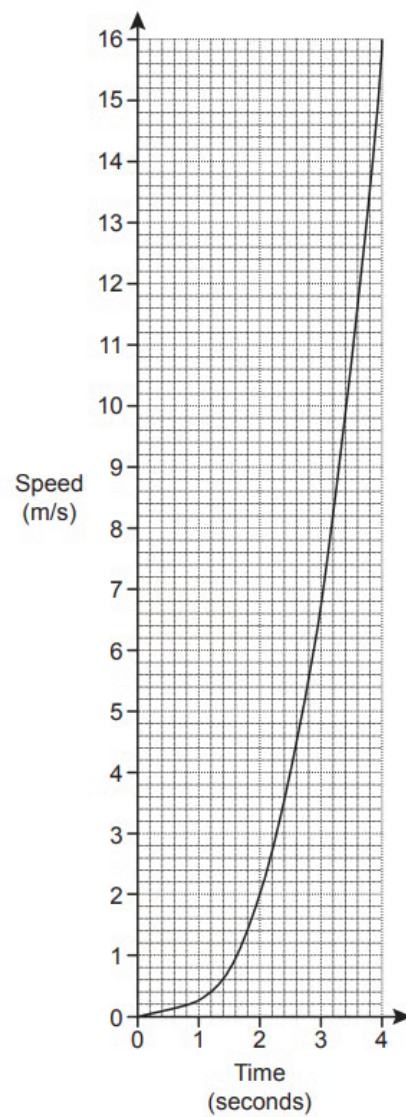
You **must** show your working.

.....cm/s

(3 marks)

12 The graph shows the speed, in metres per second, of a particle over the first four

seconds of motion.



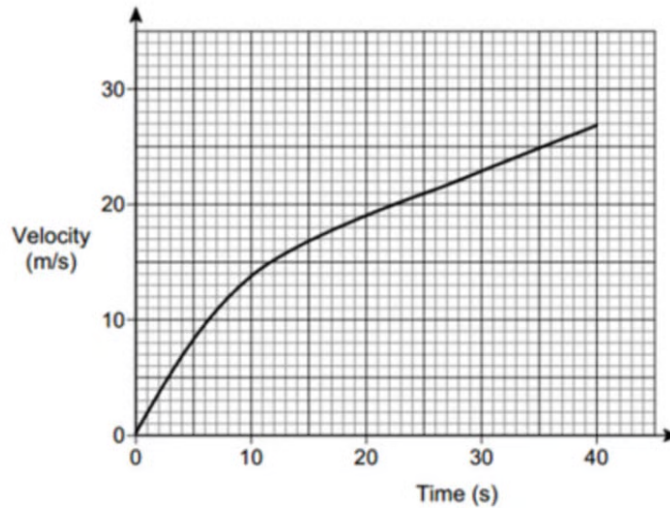
Use the graph to estimate the distance travelled by the particle in the four seconds.

..... metres

(2 marks)

Very Hard Questions

1 (a) The velocity-time graph shows the first 40 seconds of a car in a race.



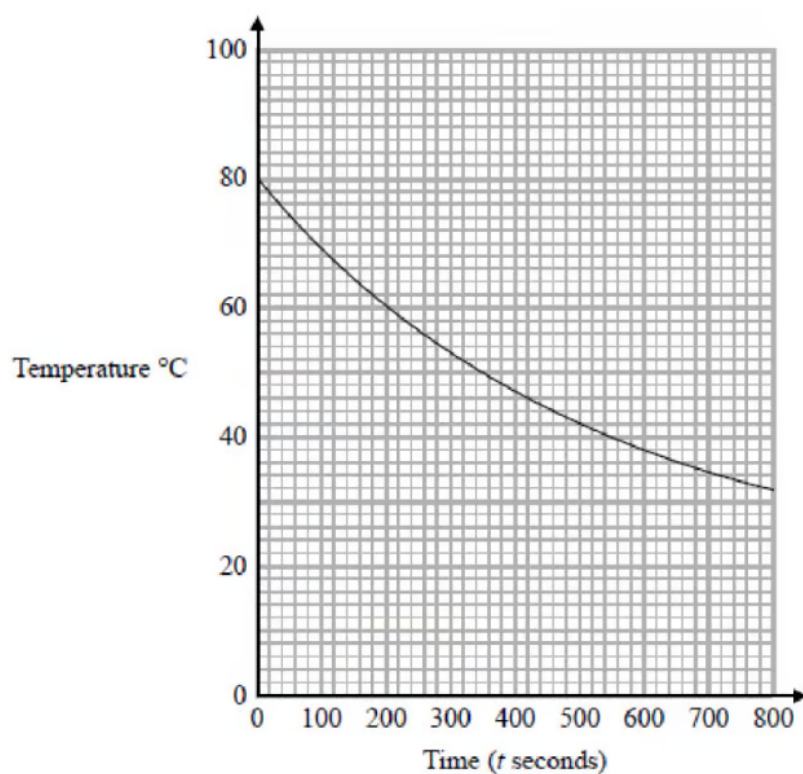
Work out the average acceleration for the first 40 seconds. Give the units of your answer.

(3 marks)

(b) Estimate the time during the 40 seconds when the instantaneous acceleration = the average acceleration. You must show your working on the graph.

(2 marks)

- 2 (a)** The graph gives information about the variation in the temperature of an amount of water that is left to cool from 80°C .



Work out an estimate for the rate of decrease of temperature at $t = 300$.

(2 marks)

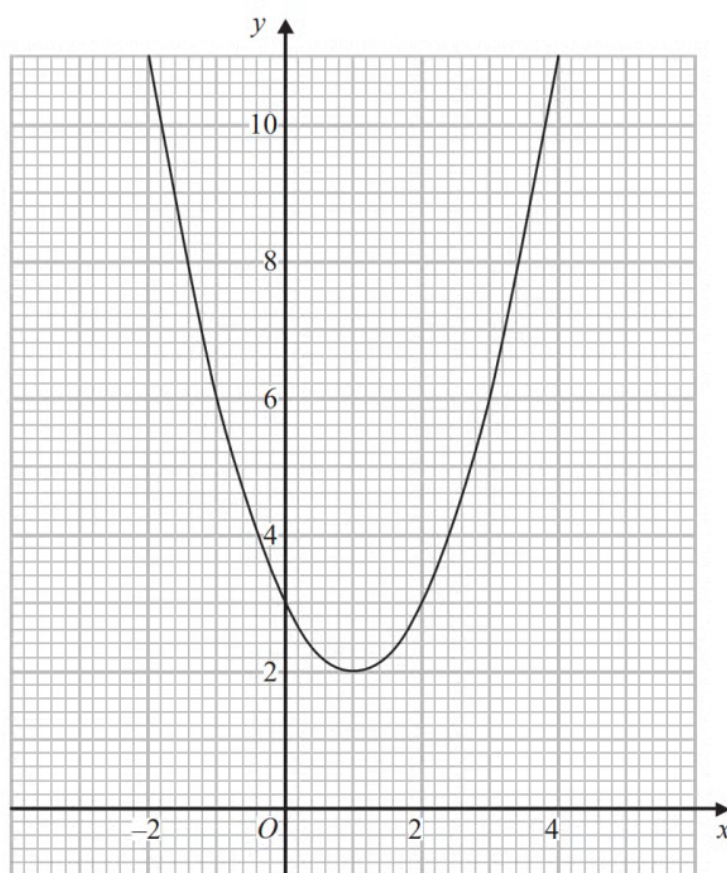
- (b)** Work out the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$.

(2 marks)

- (c)** The instantaneous rate of decrease of the temperature of the water at time T seconds is equal to the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$. Find an estimate for the value of T . You must show how you got your answer.

(2 marks)

3 (a) The diagram shows part of the graph of $y = x^2 - 2x + 3$



By drawing a suitable straight line, use your graph to find estimates for the solutions of $x^2 - 3x - 1 = 0$

(2 marks)

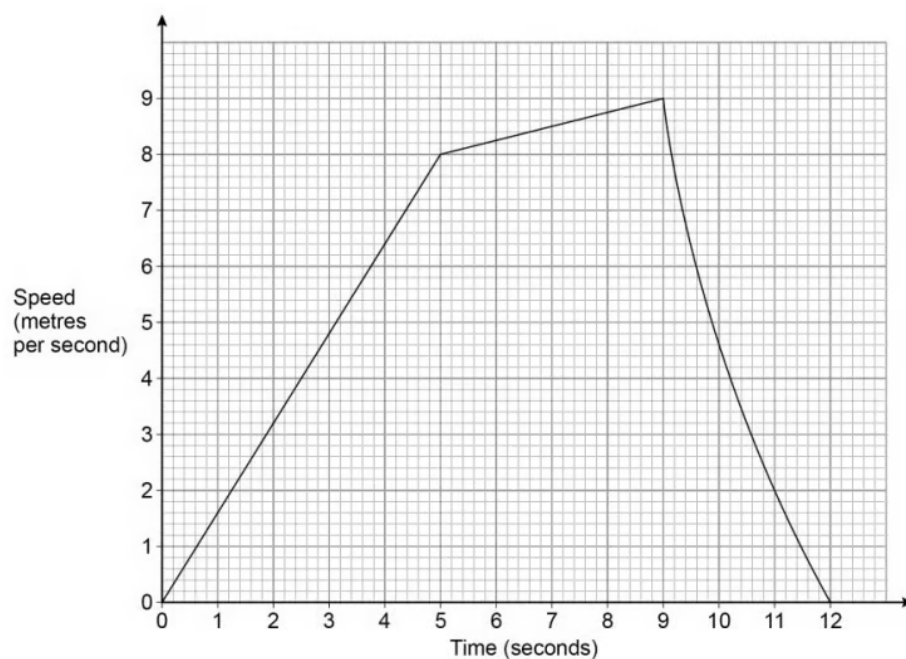
(b) P is the point on the graph of $y = x^2 - 2x + 3$ where $x = 2$

Calculate an estimate for the gradient of the graph at the point P .

(3 marks)

4 (a) Leo runs for 12 seconds.

The graph shows his speed.



Show that the distance he runs is less than 67.5 metres.

(4 marks)

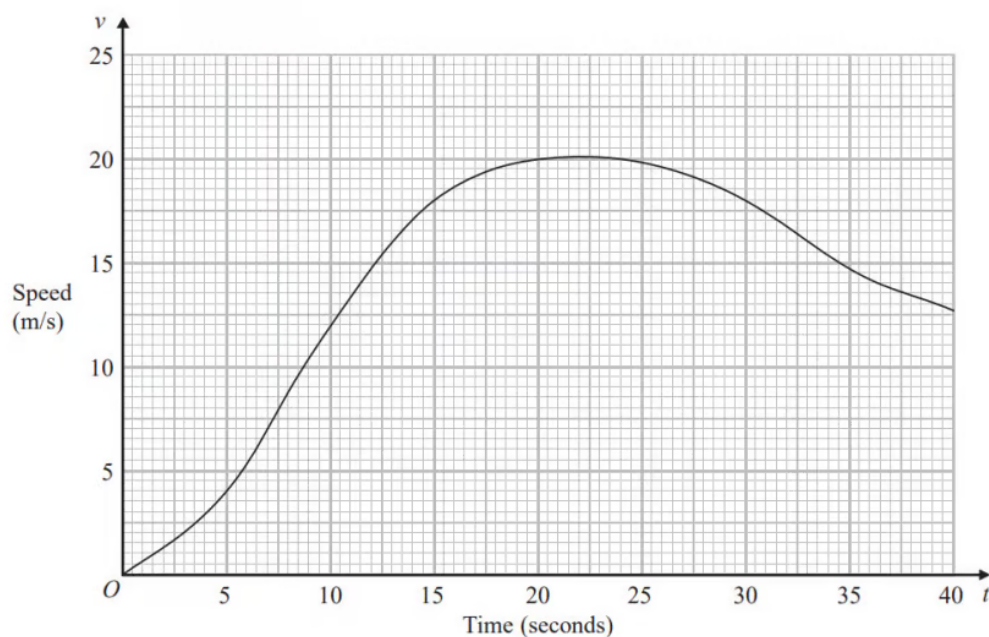
(b) Work out his average acceleration for the first 9 seconds.

State the units of your answer.

(2 marks)

5 (a) A car moves from rest.

The graph gives information about the speed, v metres per second, of the car t seconds after it starts to move.



i) Calculate an estimate of the gradient of the graph at $t = 15$

[3]

ii) Describe what your answer to part (i) represents.

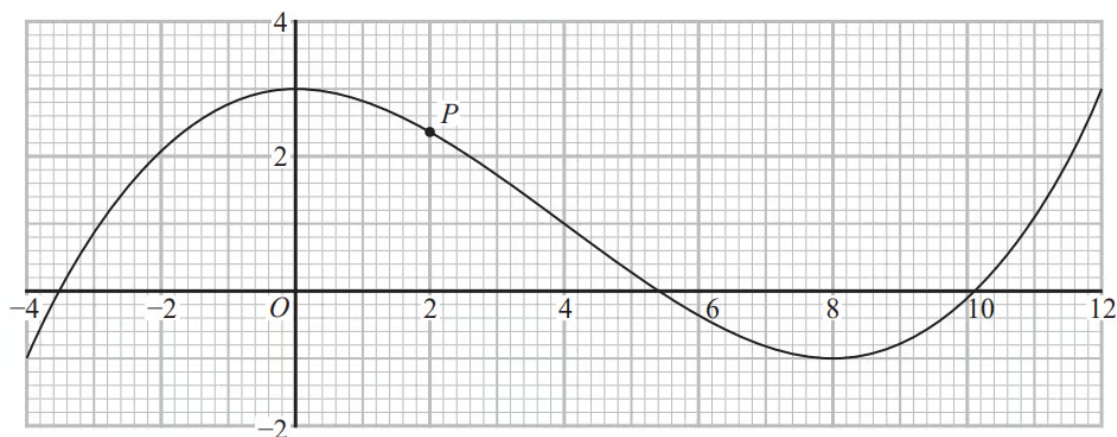
[1]

(4 marks)

(b) Work out an estimate for the distance the car travels in the first 20 seconds of its journey. Use 4 strips of equal width.

(3 marks)

6 (a) The diagram shows the graph of $y = f(x)$ for $-4 \leq x \leq 12$



The point P on the curve has x coordinate 2

Use the graph to find an estimate for the gradient of the curve at P .

(3 marks)

(b) Hence find an equation of the tangent to the curve at P . Give your answer in the form $y = mx + c$.

(2 marks)