GCSE · Edexcel · Maths

3 hours · 43 questions

Exam Questions

Algebraic Proof

Algebraic Proof

Medium (16 questions)	/51
Hard (15 questions)	/60
Very Hard (12 questions)	/44
Total Marks	**/155**

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Medium Questions

1 Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

(3 marks)

2 Show that $(n+3)^2 - (n-3)^2$ is an even number for all positive integer values of n.

(3 marks)

3 n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n-2)^2$ is always an even number.

(4 marks)

4 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(3 marks)

5 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(3 marks)

6 $E = n^2 + n + 5$

Ali thinks that the value of E will be a prime number for any whole number value of n.

Is Ali correct?
You must give a reason for your answer.

(2 marks)

7 p is a positive number.

n is a negative number.

For each statement, tick the correct box.

	Always true	Sometimes true	Never true
$p + n$ is positive	☐	☐	☐
$p - n$ is positive	☐	☐	☐
$p^2 + n^2$ is positive	☐	☐	☐
$p^3 \div n^3$ is positive	☐	☐	☐

(4 marks)

8 x is an integer.

Prove that $35 + (3x + 1)^2 - 2x(4x - 3)$ is a square number.

(4 marks)

9 Which of these is a correct identity?

A. $x + 4x \equiv 5x$

B. $6x \equiv 18$

C. $2x + 1 \equiv 7$

D. $7x + 9 \equiv x$

(1 mark)

10

$$k = n^2 + 9n + 1$$

Mo says,
"k will be a prime number for all integer values of n from 1 to 9"

Show that Mo is wrong.
You **must** show that your value of k is **not** prime.

(3 marks)

11 Tick whether the following statement is true or false.

Give a reason for your answer.

When n is a positive integer, the value of $2n$ is **always** a factor of the value of $20n$.

True ☐ False ☐

(1 mark)

12 Prove that the mean of any four consecutive even integers is an integer.

(4 marks)

13 Bethany says that $(2x)^2$ is always greater than or equal to $2x$.

Decide whether she is correct or not.
Show your working to justify your decision.

(3 marks)

14 n is a positive integer.

Prove that $13n + 3 + (3n - 5)(2n + 3)$ is a multiple of 6.

(4 marks)

15 Prove that the difference between two consecutive square numbers is always odd.

(4 marks)

16 (a) Prove that the sum of four consecutive whole numbers is always even.

(3 marks)

(b) Give an example to show that the sum of four consecutive integers is **not** always divisible by 4.

(2 marks)

Hard Questions

1 Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

(3 marks)

2 Prove that the square of an odd number is always 1 more than a multiple of 4

(4 marks)

3 Prove that , for all positive values of n,

$$\frac{(n+2)^2 - (n+1)^2}{2n^2 + 3n} = \frac{1}{n}$$

(4 marks)

4 Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(4 marks)

5 Prove algebraically that the product of any two odd numbers is always an odd number.

(4 marks)

6 (a) Show that $x(x - 1)(x + 1) = x^3 - x$

(1 mark)

(b) Prove that the difference between a whole number and the cube of this number is always a multiple of 6

(3 marks)

7 Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

(4 marks)

8 (a) Prove that $(2x + 1)(3x + 2) + x(3x + 5) + 2$ is a perfect square.

(6 marks)

(b) Gemma says

The equation $(2x + 1)(3x + 2) + x(3x + 5) + 2 = -12$ has no solutions.

Explain Gemma's reasoning.

(1 mark)

9 (a) n is an integer.

Explain why $2n + 1$ is an odd number.

(1 mark)

(b) Prove that the difference between the squares of two **consecutive** odd numbers is a multiple of 8.

(5 marks)

10 The lengths of the sides of a right-angled triangle are all integers.
Prove that if the lengths of the two shortest sides are even, then the length of the third side must also be even.

(3 marks)

11 (a) Express as a single fraction.

$$\frac{m+1}{n+1} - \frac{m}{n}$$

Simplify your answer.

(2 marks)

(b) Using your answer to part (**a**), prove that if m and n are positive integers and $m < n$,

then $\frac{m+1}{n+1} - \frac{m}{n} > 0$

(2 marks)

12 n is the middle integer of three consecutive positive integers.

The three integers are multiplied to give a product.

n is then added to the product.

Prove that the result is a cube number.

(4 marks)

13 Expressions for consecutive triangular numbers are

$$\frac{n(n+1)}{2} \text{ and } \frac{(n+1)(n+2)}{2}$$

Prove that the sum of two consecutive triangular numbers is always a square number.

(4 marks)

14 n is a positive integer.

Prove algebraically that $2n^2\left(\frac{3}{n} + n\right) + 6n\left(n^2 - 1\right)$ is a cube number.

(3 marks)

15 $a^2 - b^2 \equiv (a + b)(a - b)$

- a and b are positive whole numbers with $a > b$
- $a^2 - b^2$ is a **prime** number.

Why are a and b consecutive numbers?

(2 marks)

Very Hard Questions

1 i) Factorise $2t^2 + 5t + 2$

[2]

ii) t is a positive whole number.

The expression $2t^2 + 5t + 2$ can never have a value that is a prime number.

Explain why.

[1]

(3 marks)

2 n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 1)(n + 2)$ is always a square number.

(2 marks)

3 Here are the first five terms of an arithmetic sequence.

7 13 19 25 31

Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24.

(6 marks)

4 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

(2 marks)

5 The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(3 marks)

6 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

(3 marks)

7 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(3 marks)

8 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(3 marks)

9 (a) Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers $1\ 2\ 3\ 4\ \ldots$
The denominators of the fractions form the sequence of odd numbers $1\ 3\ 5\ 7\ \ldots$

Write down an expression, in terms of n, for the nth term of this sequence of fractions.

(2 marks)

(b) Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

(3 marks)

10 The table gives information about the first six terms of a sequence of numbers.

Term number	1	2	3	4	5	6
Term of sequence	$\frac{1 \times 2}{2}$	$\frac{2 \times 3}{2}$	$\frac{3 \times 4}{2}$	$\frac{4 \times 5}{2}$	$\frac{5 \times 6}{2}$	$\frac{6 \times 7}{2}$

Prove algebraically that the sum of any two consecutive terms of this sequence is always a square number.

(4 marks)

11 Prove that $x^2 + x + 1$ is always positive.

(3 marks)

12 (a) The diagram shows a cross placed on a number grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

L is the product of the left and right numbers of the cross.
T is the product of the top and bottom numbers of the cross.
M is the middle number of the cross.

Show that when $M = 35$, $L - T = 99$.

(2 marks)

(b) Prove that, for any position of the cross on the number grid above, $L - T = 99$.

(5 marks)