

Further Graphs & Tangents

Types of Graphs / Drawing Graphs from Tables / Solving Equations from Graphs / Finding Gradients of Tangents

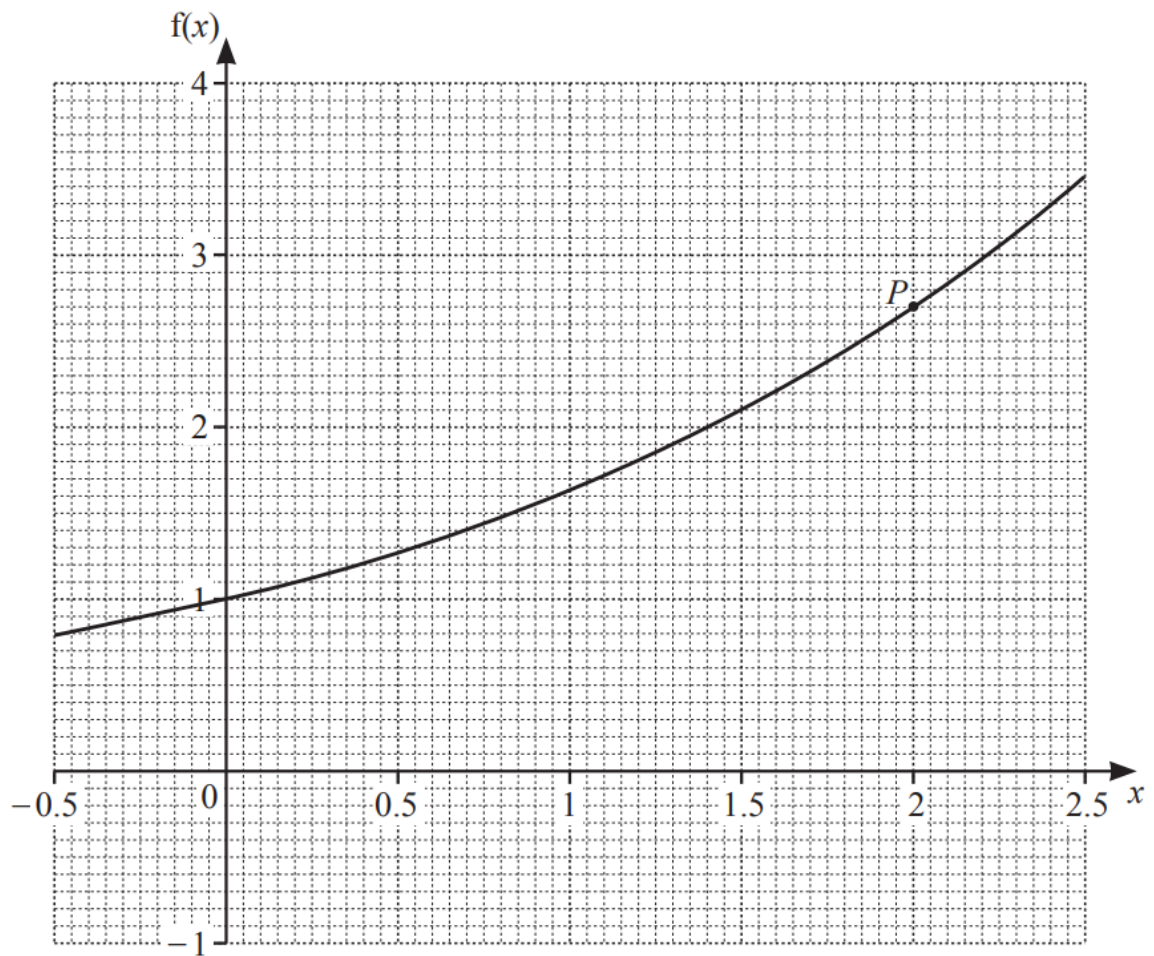
Medium (7 questions)	/32
Hard (6 questions)	/83
Very Hard (4 questions)	/54
Total Marks	/169

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Medium Questions

1

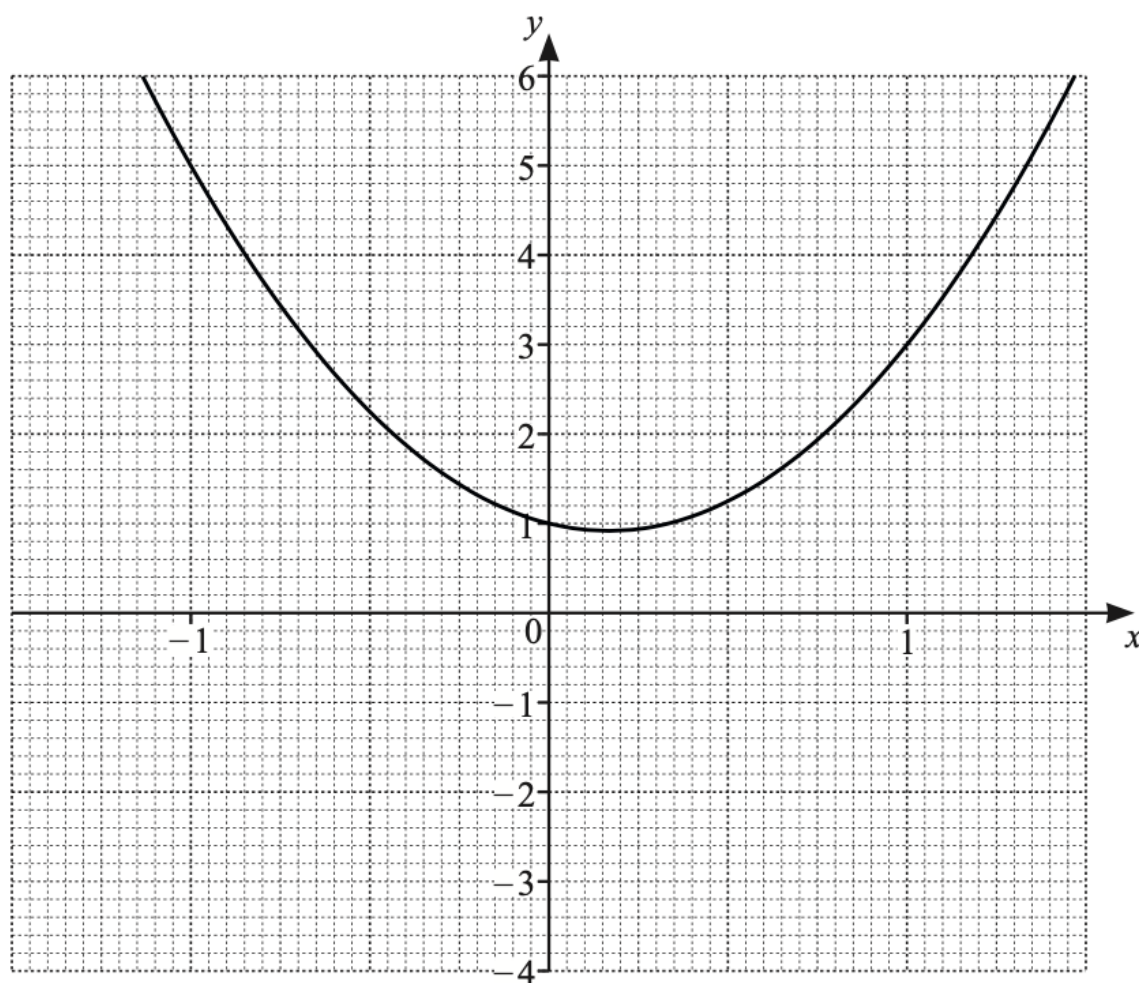


The diagram shows the graph of a function.

By drawing a suitable tangent, find an estimate for the gradient of the function at the point P .

(3 marks)

2 (a)



i) Draw the tangent to the curve at $x = 1$.

[1]

ii) Use your tangent to estimate the gradient of the curve at $x = 1$.

[2]

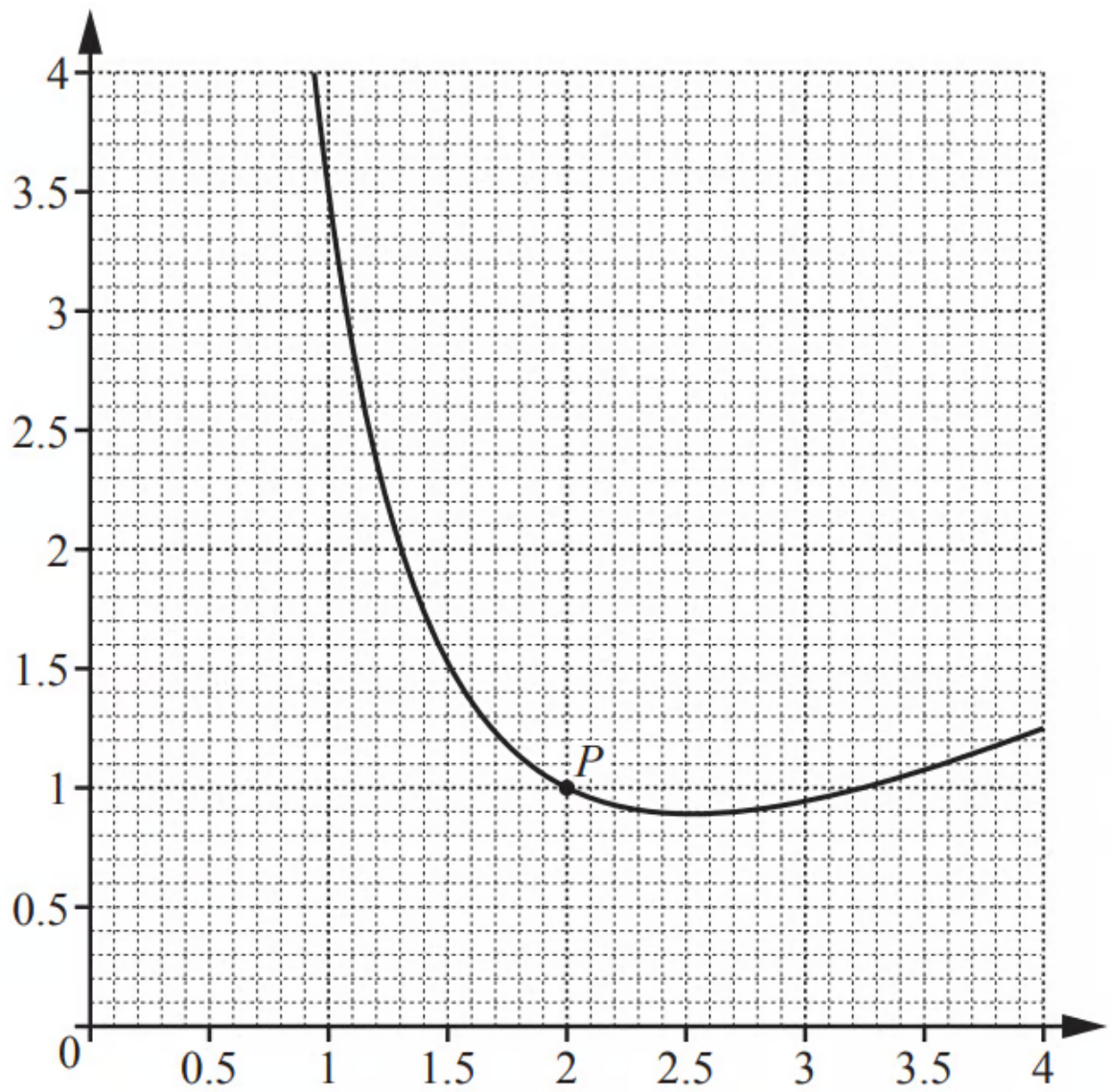
(3 marks)

(b) Write down the equation of your tangent in the form $y = mx + c$.

$y =$

(2 marks)

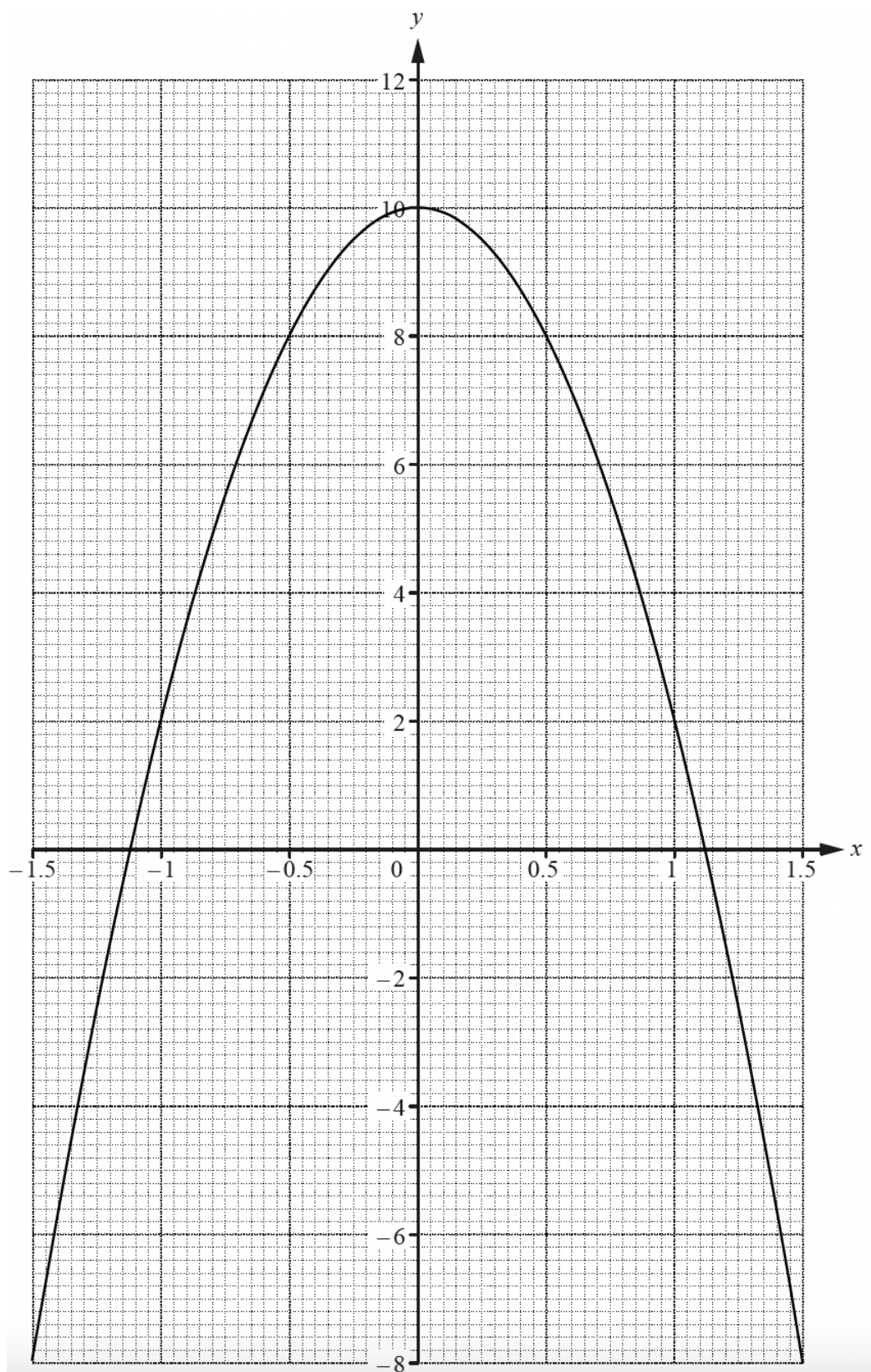
3



By drawing a suitable tangent, estimate the gradient of the curve at the point P .

(3 marks)

4 (a) The graph of $y = 10 - 8x^2$ for $-1.5 \leq x \leq 1.5$ is drawn on the grid.



Write down the equation of the line of symmetry of the graph.

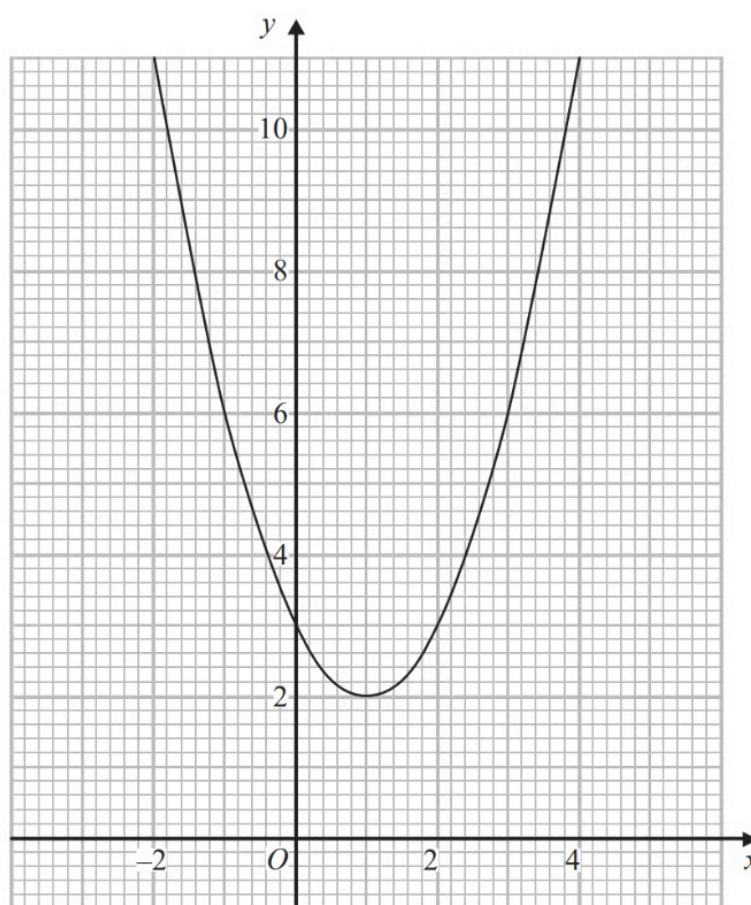
(1 mark)

(b) On the grid, draw the tangent to the curve at the point where $x = 0.5$.

Find the gradient of this tangent.

(3 marks)

5 (a) The diagram shows part of the graph of $y = x^2 - 2x + 3$



By drawing a suitable straight line, use your graph to find estimates for the solutions of $x^2 - 3x - 1 = 0$

(2 marks)

(b) P is the point on the graph of $y = x^2 - 2x + 3$ where $x = 2$

Calculate an estimate for the gradient of the graph at the point P .

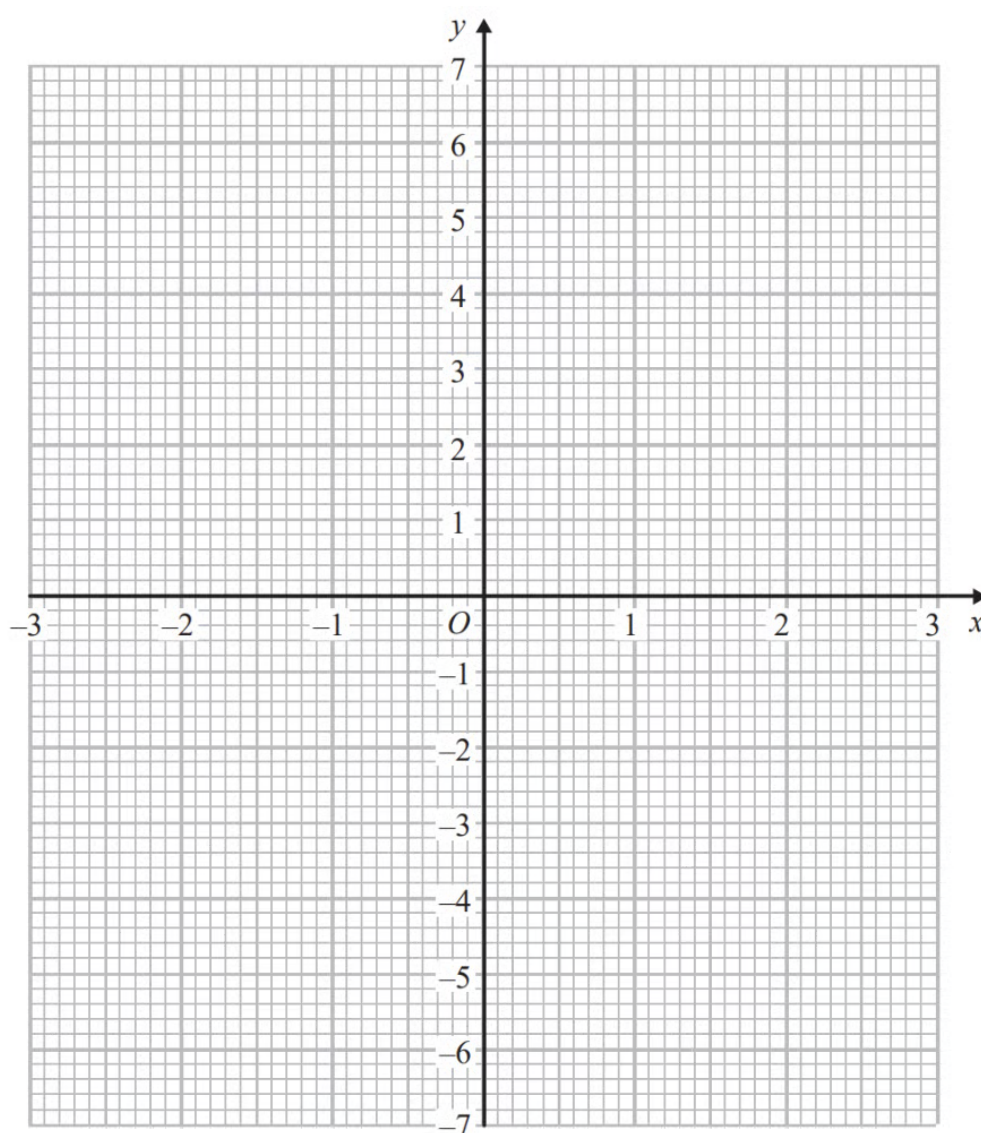
(3 marks)

6 (a) Complete the table of values for $y = x^2 - x - 6$

x	-3	-2	-1	0	1	2	3
y	6			-6			

(2 marks)

(b) On the grid, draw the graph of $y = x^2 - x - 6$ for values of x from -3 to 3



(2 marks)

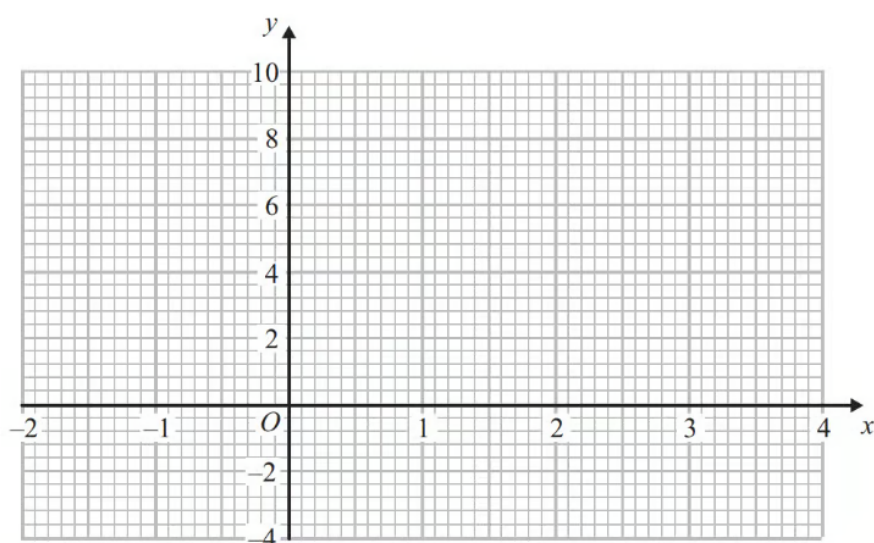
- (c) Use your graph to find estimates of the solutions to the equation $x^2 - x - 6 = -2$
(2 marks)

7 (a) Complete the table of values for $y = x^2 - 2x$

x	-2	-1	0	1	2	3	4
y		3	0			3	

(2 marks)

(b) On the grid, draw the graph of $y = x^2 - 2x$ for values of x from -2 to 4



(2 marks)

(c) Solve $x^2 - 2x - 2 = 1$

(2 marks)

Hard Questions

1 (a) $y = x^2 + \frac{1}{x}, \quad x \neq 0$

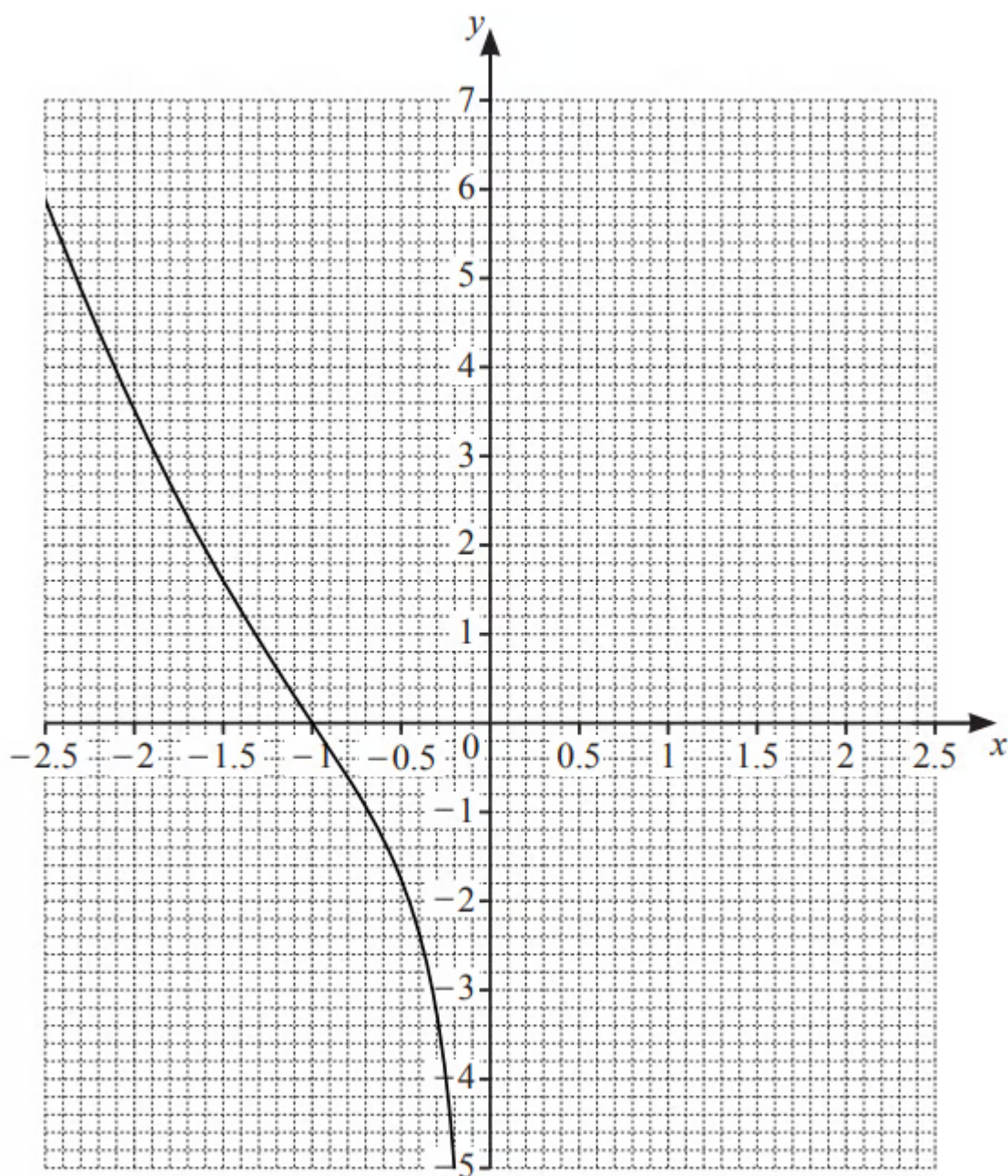
Complete the table.

x	0.2	0.3	0.5	1	1.5	2	2.5
y	5.0	3.4	2.3		2.9		6.7

(2 marks)

(b) On the grid, draw the graph of $y = x^2 + \frac{1}{x}$ for $0.2 \leq x \leq 2.5$.

The graph of $y = x^2 + \frac{1}{x}$ for $-2.5 \leq x \leq -0.2$ has been drawn for you.



(4 marks)

(c) By drawing suitable straight lines on the grid, solve the following equations.

i) $x^2 + \frac{1}{x} = -2$

$$x = \dots\dots\dots [1]$$

ii) $x^2 + \frac{1}{x} + x - 1 = 0$

$$x = \dots\dots\dots [2]$$

(3 marks)

- (d)** k is an integer and the equation $x^2 + \frac{1}{x} = k$ has three solutions. Write down a possible value of k .

$$k = \dots\dots\dots$$

(1 mark)

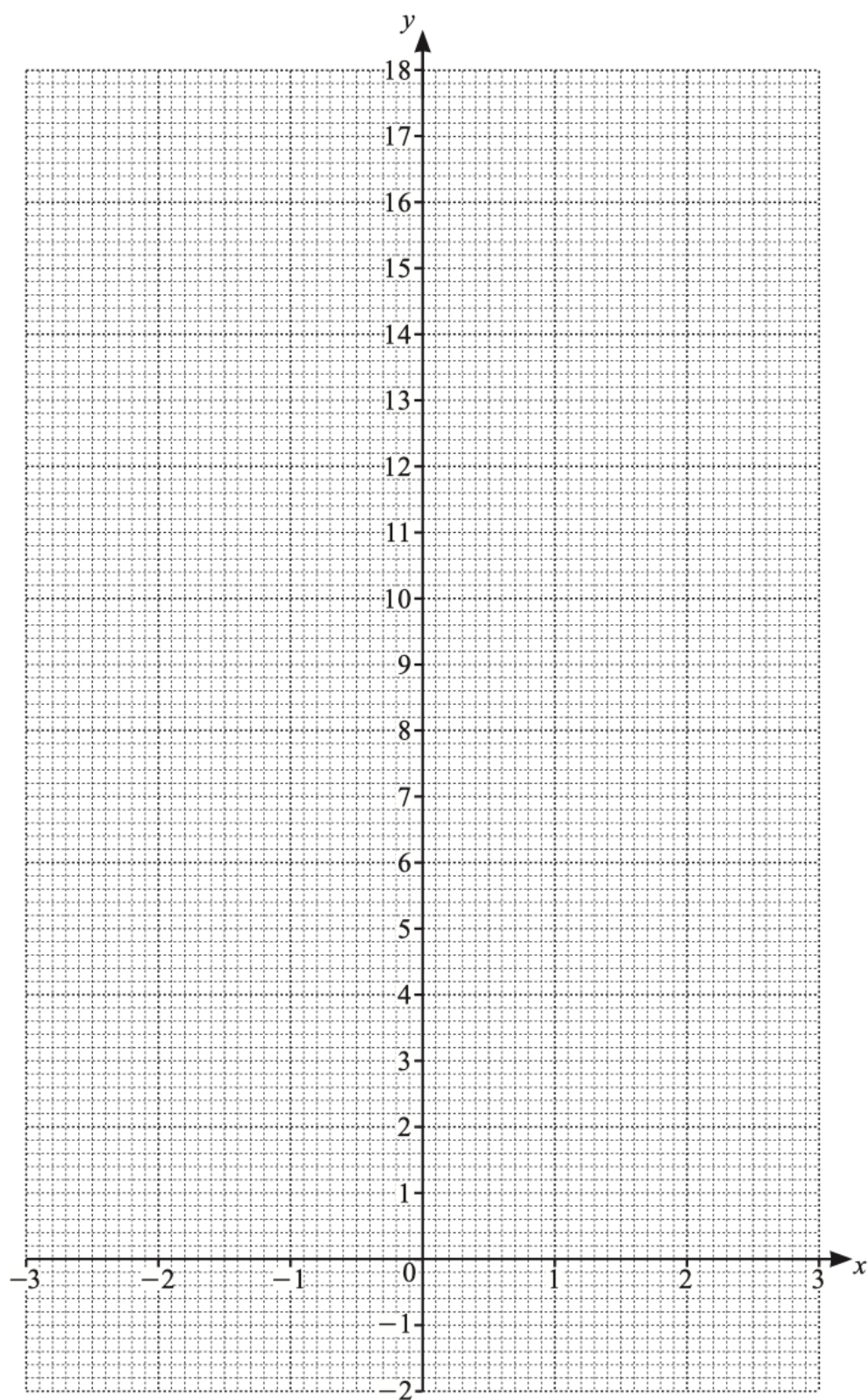
2 (a) The table shows some values of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.3		0.2	0.3	0.5	1	2	3
y	5.3	3.3		8.1	17.8			4.5	0.1	-0.5	1.3	

Complete the table.

(3 marks)

(b) On the grid, draw the graph of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}$ for $-3 \leq x \leq -0.3$ and $0.2 \leq x \leq 3$.



(5 marks)

(c) Use your graph to solve $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} \leq 0$.

..... $\leq x \leq$

(2 marks)

(d) Find the smallest positive integer value of k for which $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = k$ has two solutions for $-3 \leq x \leq -0.3$ and $0.2 \leq x \leq 3$.

(1 mark)

(e) By drawing a suitable straight line, solve $\frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x} = 3x + 1$ for $-3 \leq x \leq -0.3$ and $0.2 \leq x \leq 3$.

$x =$

(3 marks)

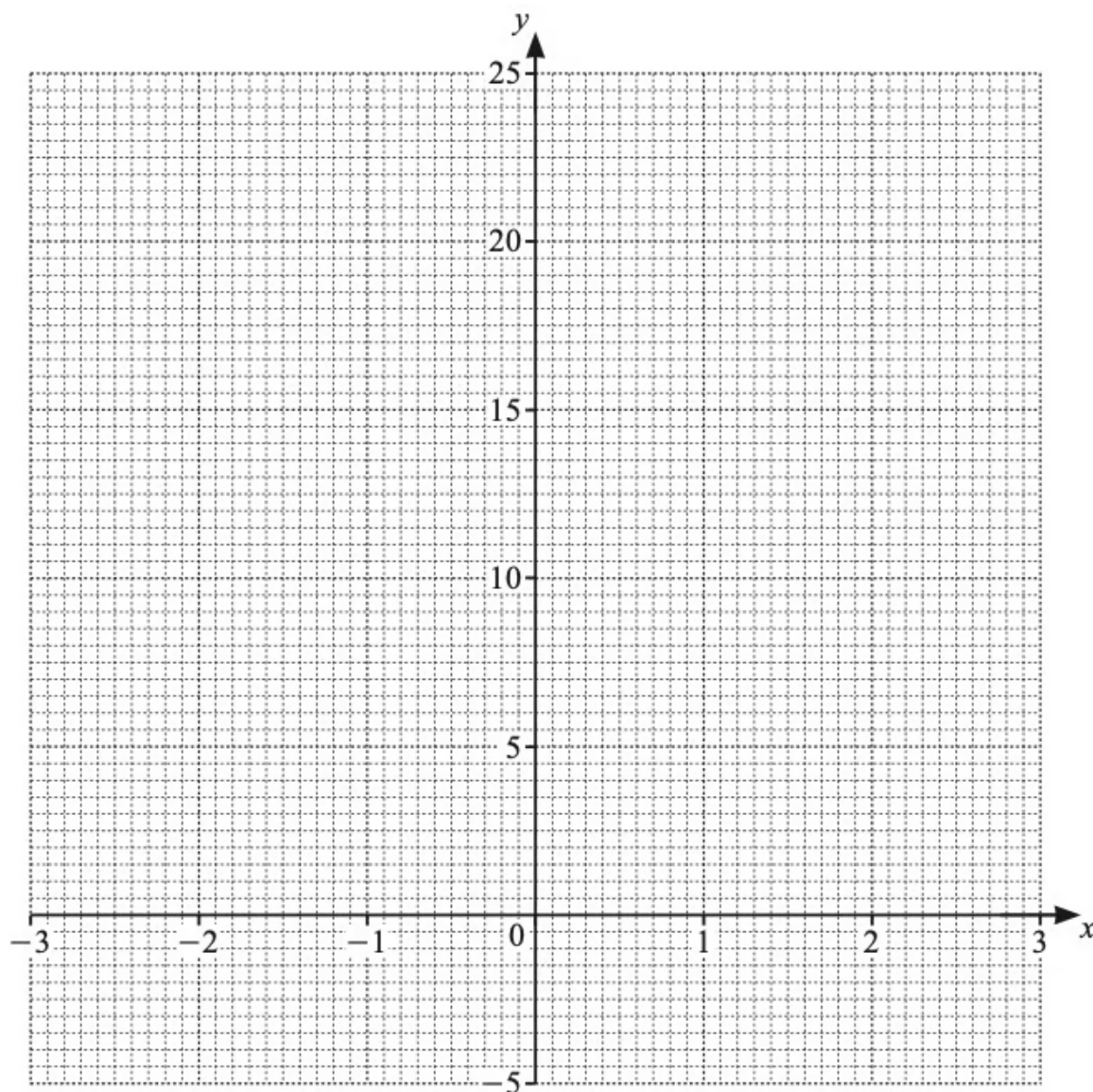
3 (a) The table shows some values for $y = x^3 + x^2 - 5x$.

x	-3	-2	-1.5	-1	0	1	1.5	2	2.5	3
y	-3	6	6.4		0		-1.9	2	9.4	

Complete the table.

(3 marks)

(b) On the grid, draw the graph of $y = x^3 + x^2 - 5x$ for $-3 \leq x \leq 3$.



(4 marks)

(c) Use your graph to solve the equation $x^3 + x^2 - 5x = 0$.

$x = \dots\dots\dots$ or $x = \dots\dots\dots$ or $x = \dots\dots\dots$

(2 marks)

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = 2$.
(3 marks)

(e) Write down the largest value of the integer, k , so that the equation $x^3 + x^2 - 5x = k$ has three solutions for $-3 \leq x \leq 3$.

$k = \dots\dots\dots$

(1 mark)

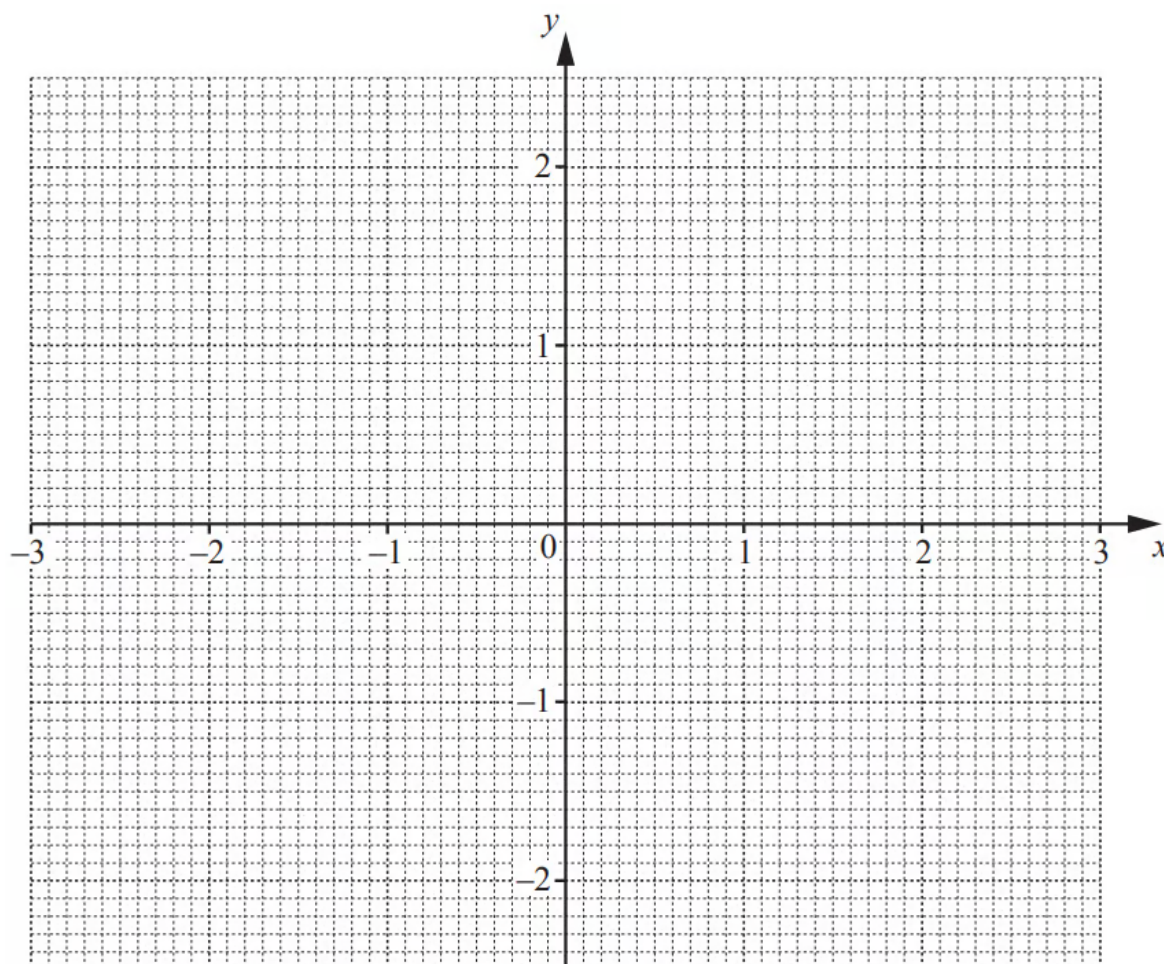
- 4 (a)** The table shows some values for $y = \frac{3}{10}x^3 - 2x$ for $-3 \leq x \leq 3$.

x	-3	-2	-1.5	-1	0	1	1.5	2	3
y			2.0	1.7	0		-2.0	-1.6	

Complete the table.

(3 marks)

- (b)** On the grid, draw the graph of $y = \frac{3}{10}x^3 - 2x$ for $-3 \leq x \leq 3$.



(4 marks)

- (c) On the grid, draw a suitable straight line to solve the equation $\frac{3}{10}x^3 - 2x = \frac{1}{2}(1 - x)$ for $-3 \leq x \leq 3$.

$x = \dots\dots\dots$ or $x = \dots\dots\dots$ or $x = \dots\dots\dots$

(4 marks)

- (d) For $-3 \leq x \leq 3$, the equation $\frac{3}{10}x^3 - 2x = 1$ has n solutions. Write down the value of n .

$n = \dots\dots\dots$

(1 mark)

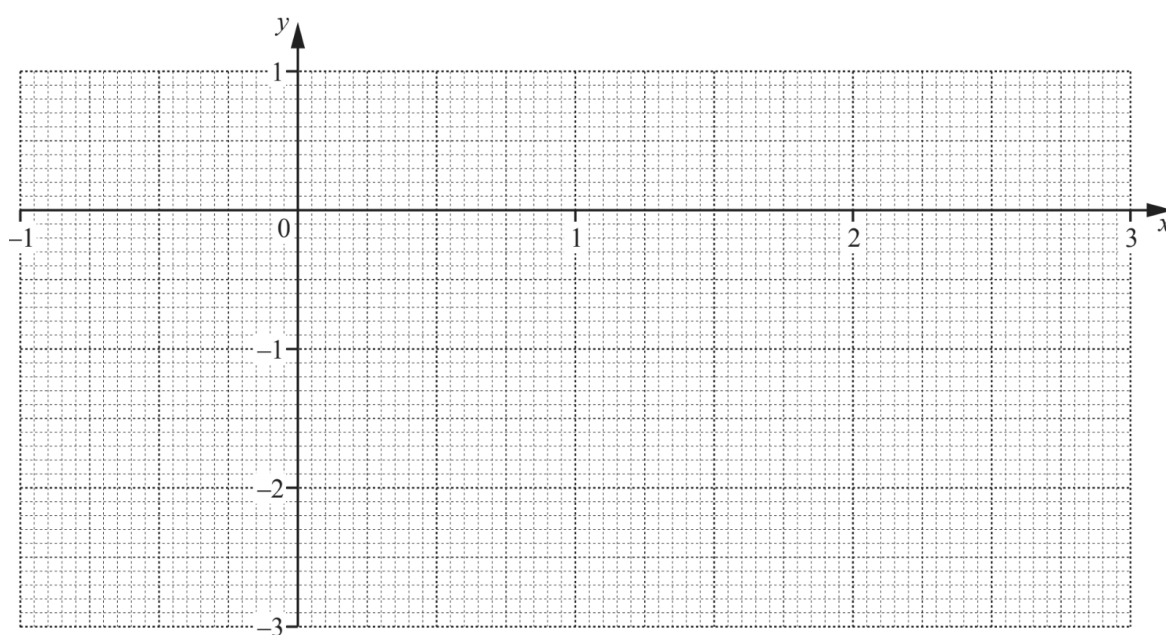
5 (a) The table shows some values of $y = x^3 - 3x^2 + x$.

x	-0.75	-0.5	-0.25	0	0.5	1	1.5	2	2.5	2.75
y	-2.9	-1.4	-0.5		-0.1	-1	-1.9		-0.6	

Complete the table.

(3 marks)

(b) On the grid, draw the graph of $y = x^3 - 3x^2 + x$ for $-0.75 \leq x \leq 2.75$.



(3 marks)

(c) Use your graph to complete the inequalities in x for which $y > -1$.

..... $< x <$ and $x >$
(3 marks)

(d) The equation $x^3 - 3x^2 + 2x - 1 = 0$ can be solved by drawing a straight line on the grid.

i) Write down the equation of this line.

[2]

ii) On the grid, draw this line and use it to solve the equation $x^3 - 3x^2 + 2x - 1 = 0$.

$x =$ [3]
(5 marks)

(e) By drawing a suitable tangent, find an estimate for the gradient of the graph of $y = x^3 - 3x^2 + x$ at $x = -0.25$.

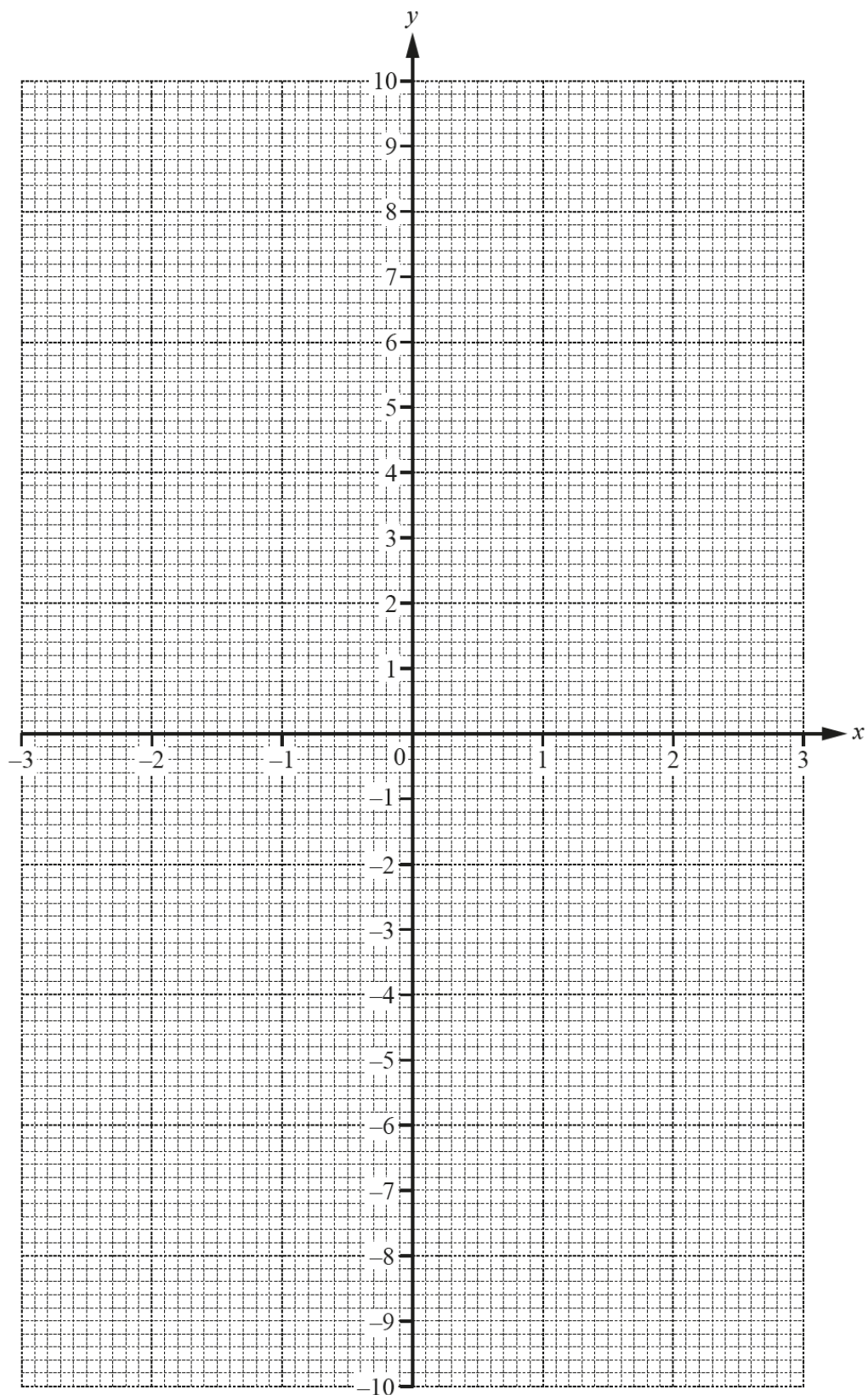
(3 marks)

6 (a) Complete the table of values for $y = \frac{x^3}{3} - \frac{1}{2x^2}$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
y	-9.1	-2.8	-0.8		-5.6		-5.5	-2.0			8.9

(3 marks)

(b) On the grid, draw the graph of $y = \frac{x^3}{3} - \frac{1}{2x^2}$ for $-3 \leq x \leq -0.3$ and $0.3 \leq x \leq 3$.



(5 marks)

- (c) i) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$.

[3]

- ii) Write down the equation of the tangent to the curve at $x = -2$. Give your answer in the form $y = mx + c$.

$y = \dots\dots\dots$ [2]

(5 marks)

- (d) Use your graph to solve the equations.

i) $\frac{x^3}{3} - \frac{1}{2x^2} = 0$

$x = \dots\dots\dots$ [1]

ii) $\frac{x^3}{3} - \frac{1}{2x^2} + 4 = 0$

$x = \dots\dots\dots$ or $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

(4 marks)

Very Hard Questions

- 1 (a)** The table shows some values of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leq x \leq 3.5$.

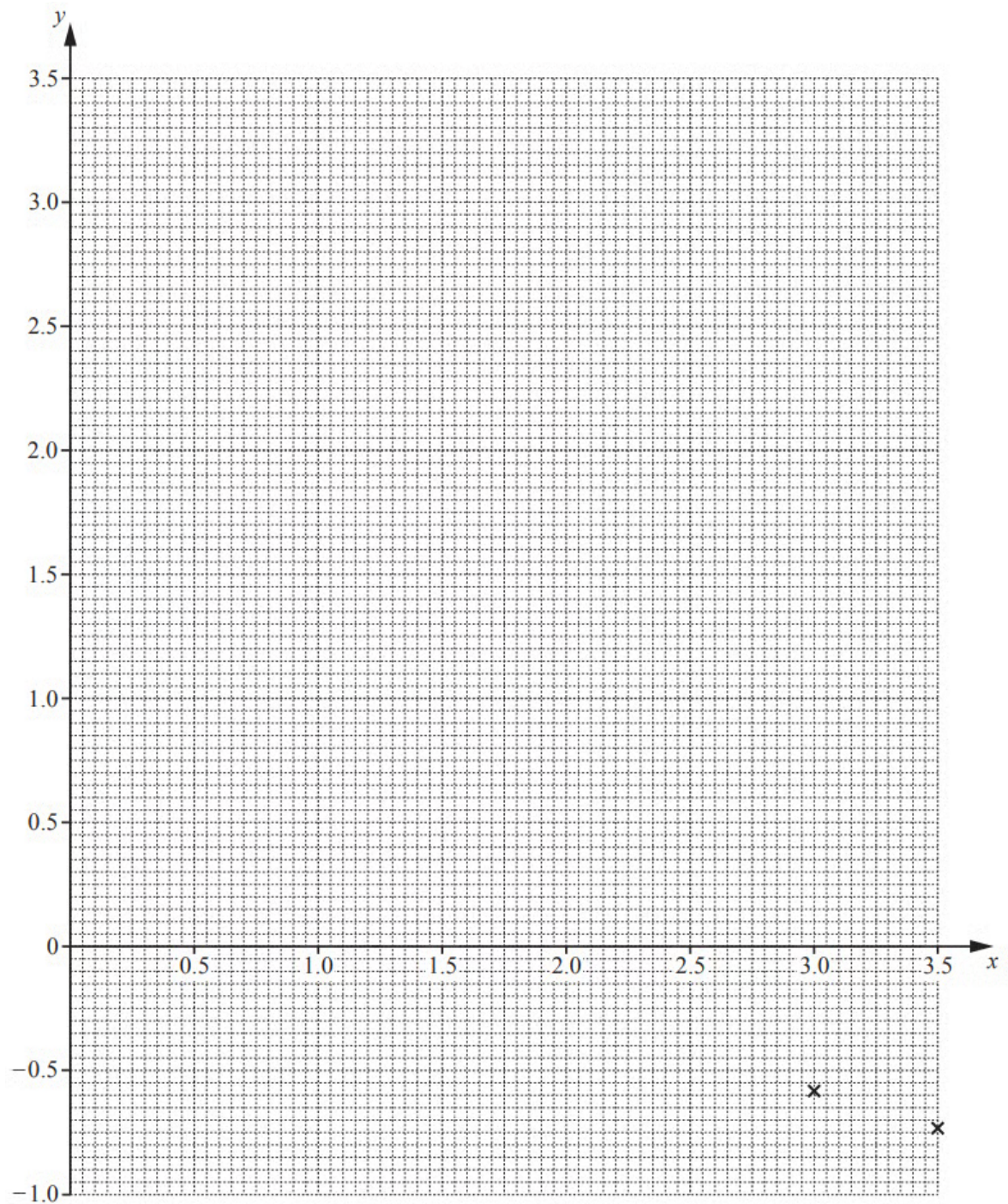
x	0.15	0.2	0.5	1	1.5	2	2.5	3	3.5
y	3.30		0.88		-0.04		-0.43	-0.58	-0.73

Complete the table.

(3 marks)

- (b)** On the grid, draw the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leq x \leq 3.5$. The last two points have

been plotted for you.



(4 marks)

(c) Use your graph to solve the equation $\frac{1}{2x} - \frac{x}{4} = \frac{1}{2}$ for $0.15 \leq x \leq 3.5$.

$x = \dots\dots\dots$

(1 mark)

(d) i) On the grid, draw the line $y = 2 - x$.

[2]

ii) Write down the x co-ordinates of the points where the line $y = 2 - x$ crosses the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leq x \leq 3.5$.

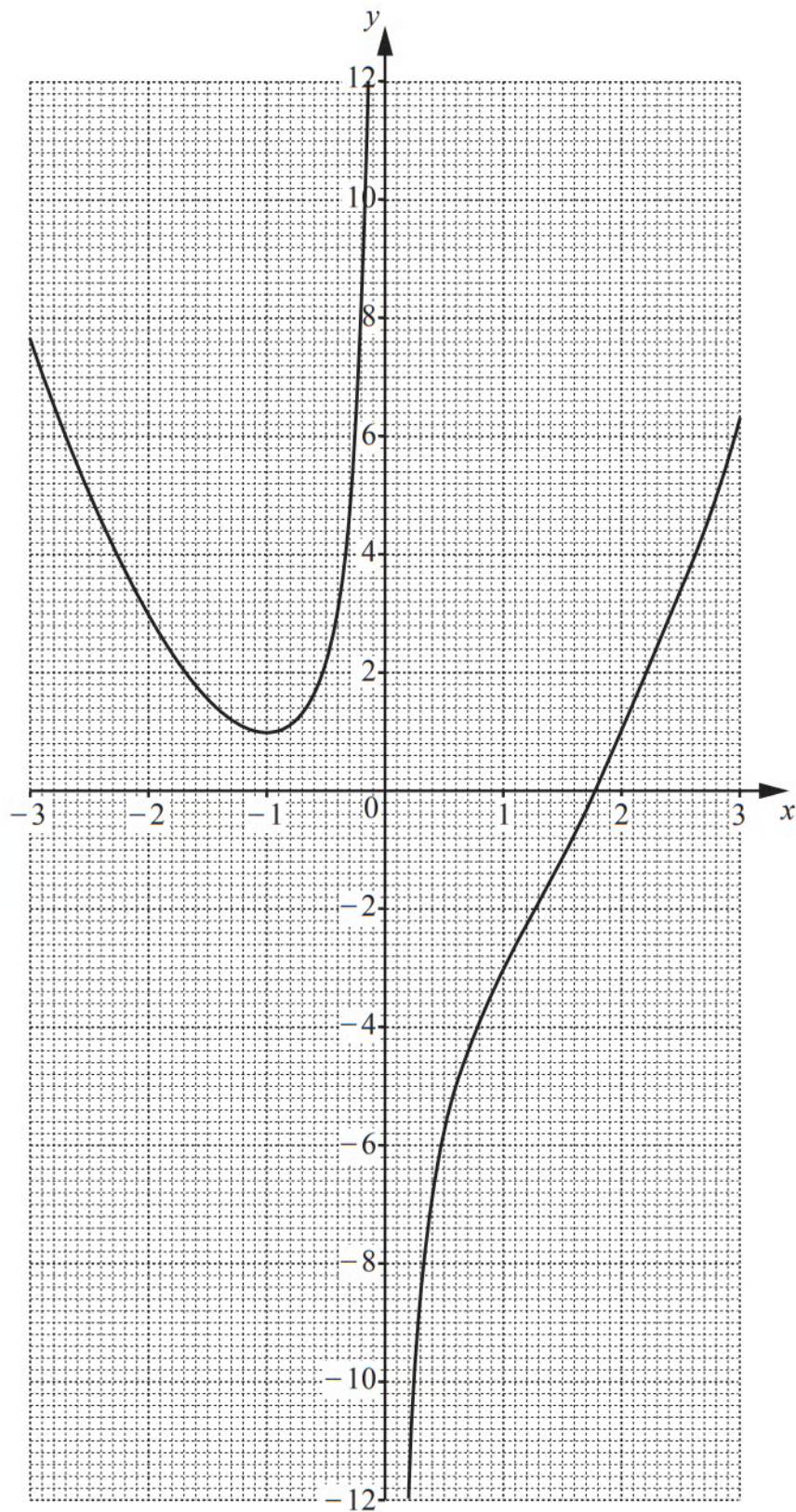
$x = \dots\dots\dots$ and $x = \dots\dots\dots$ [2]

(4 marks)

(e) Show that the graph of $y = \frac{1}{2x} - \frac{x}{4}$ can be used to find the value of $\sqrt{2}$ for $0.15 \leq x \leq 3.5$.

(2 marks)

2 (a) The diagram shows the graph of $y = f(x)$ where $f(x) = x^2 - \frac{2}{x} - 2$, $x \neq 0$.



On the grid, draw a suitable straight line to solve the equation $x^2 - \frac{2}{x} - 7 = -3x$ for $-3 \leq x \leq 3$.

$x = \dots\dots\dots$ or $x = \dots\dots\dots$
(4 marks)

(b) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$.
(3 marks)

(c) i) Complete the table for $y = g(x)$ where $g(x) = 2^{-x}$ for $-3 \leq x \leq 3$.

x	-3	-2	-1	0	1	2	3
y			2	1	0.5		0.125

[3]

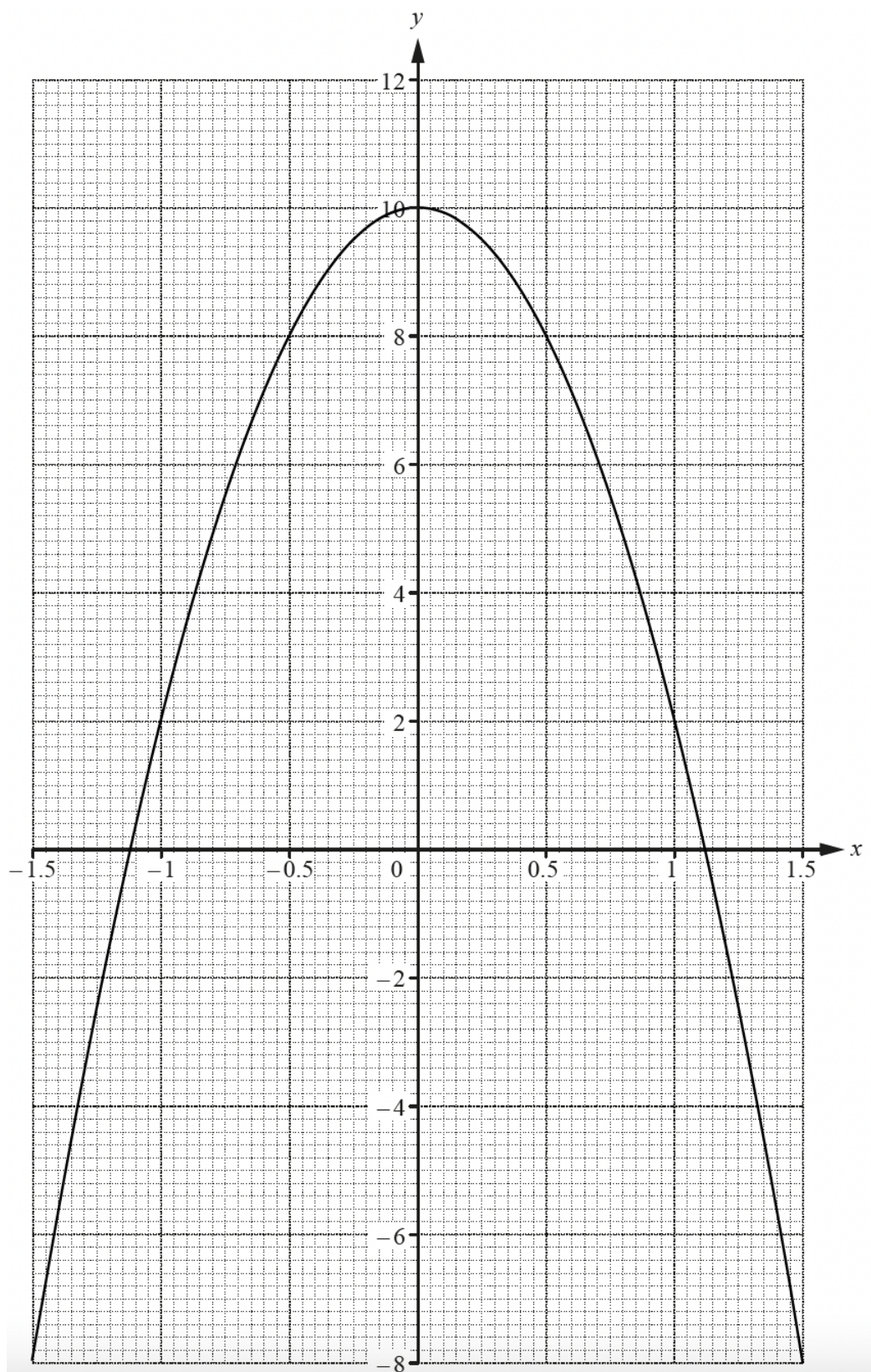
ii) On the grid, draw the graph of $y = g(x)$.

[3]

iii) Use your graph to find the **positive** solution to the equation $f(x) = g(x)$.

$x = \dots\dots\dots$ [1]
(7 marks)

3 (a) The graph of $y = 10 - 8x^2$ for $-1.5 \leq x \leq 1.5$ is drawn on the grid.



And the table shows some values for $y = x^3 + 3x + 4$.

x	-1.5	-1	-0.5	0	0.5	1	1.5
y	-3.9				5.6	8	11.9

i) Complete the table.

[3]

ii) On the grid, draw the graph of $y = x^3 + 3x + 4$ for $-1.5 \leq x \leq 1.5$.

[4]

(7 marks)

(b) Show that the values of x where the two curves intersect are the solutions to the equation $x^3 + 8x^2 + 3x - 6 = 0$.

(1 mark)

(c) By drawing a suitable straight line, solve the equation $x^3 + 5x + 2 = 0$ for $-1.5 \leq x \leq 1.5$.

$x = \dots\dots\dots$

(3 marks)

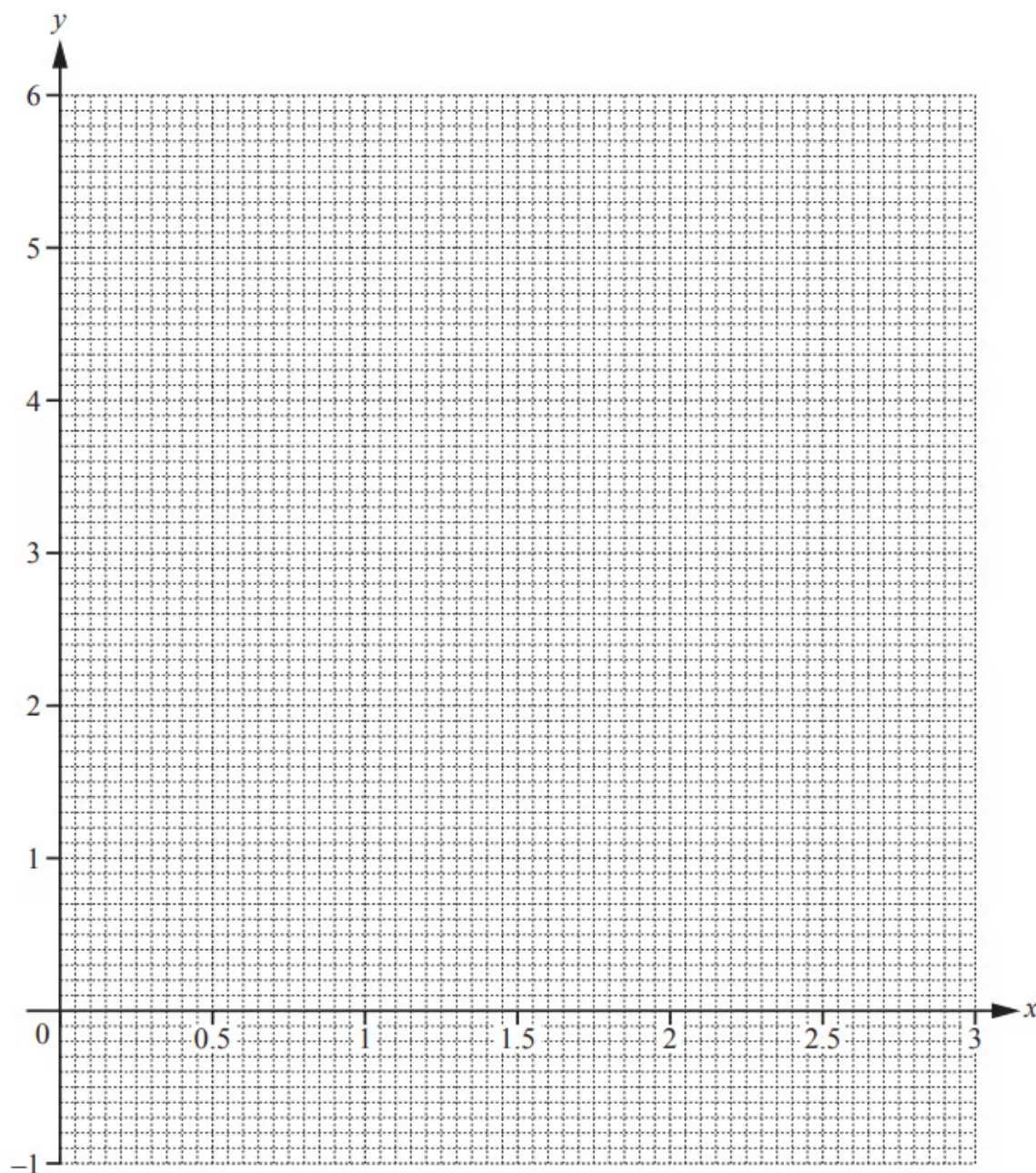
4 (a) The table shows some values for $y = 2x + \frac{1}{x} - 3$ for $0.125 \leq x \leq 3$.

x	0.125	0.25	0.375	0.5	0.75	1	1.5	2	2.5	3
y	5.25	1.5	0.42			0	0.67	1.5		3.33

Complete the table.

(3 marks)

(b) On the grid, draw the graph of $y = 2x + \frac{1}{x} - 3$ for $0.125 \leq x \leq 3$.



(4 marks)

(c) Use your graph to solve $2x + \frac{1}{x} - 3 \geq 2$.

.....

(3 marks)

(d) The equation $\frac{1}{x} = 7 - 3x$ can be solved using your graph in **part (b)** and a straight line.

i) Write down the equation of this straight line.

[2]

ii) Draw this straight line and solve the equation $\frac{1}{x} = 7 - 3x$.

$x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

(5 marks)