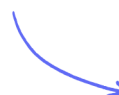


Functions

Introduction to Functions / Composite Functions / Inverse Functions

Medium (10 questions)	/48
Hard (14 questions)	/53
Very Hard (9 questions)	/33
Total Marks	/134

Scan here to return to the course
or visit [savemyexams.com](https://www.savemyexams.com)



Medium Questions

1 (a) f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

Find $f(-5)$

(1 mark)

(b) Find $fg(1)$

(2 marks)

2 (a)

$$f(x) = 3x - 2$$
$$g(x) = \frac{10}{x+2}$$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

(2 marks)

(b) Find $gf(x)$
Simplify your answer.

(2 marks)

3 (a) f is a function such that

$$f(x) = \frac{1}{x^2 + 1}$$

Find $f\left(\frac{1}{2}\right)$

(1 mark)

(b) g is a function such that

$$g(x) = \sqrt{x-1} \quad x \geq 1$$

Find $fg(x)$

Give your answer as simply as possible.

(2 marks)

4 (a) f is the function $f(x) = 2x + 5$

Find $f(3)$

(1 mark)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

(2 marks)

(c) g is the function $g(x) = x^2 - 25$

Find $g(-3)$

(1 mark)

(d) i) Find $gf(x)$

Give your answer as simply as possible.

[3]

ii) Solve $gf(x) = 0$

[2]

(5 marks)

5 (a) The functions f and g are defined as

$$f(x) = \frac{1}{2}x + 4$$

$$g(x) = \frac{2x}{x+1}$$

Work out $f(6)$

(1 mark)

(b) Work out $fg(-3)$

(2 marks)

(c) $g(a) = -2$

Work out the value of a .

(2 marks)

(d) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

(3 marks)

- 6 (a)** f is the function such that $f(x) = 2x - 5$
 g is the function such that $g(x) = x^2 - 10$

Find $f(4)$

(1 mark)

- (b)** Find $fg(-4)$

(2 marks)

- (c)** Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

(2 marks)

- (d)** Solve $gf(x) = -1$

(4 marks)

7 (a) The functions f and g are such that

$$f(x) = 3(x - 4) \text{ and } g(x) = \frac{x}{5} + 1$$

Find the value of $f(10)$

(1 mark)

(b) Find $g^{-1}(x)$

(2 marks)

(c) Show that $ff(x) = 9x - 48$

(2 marks)

8 $g(x) = 2x$ and $h(x) = \frac{x-1}{2}$

Choose the expression for $hg(x)$

A. $\frac{2x^2 - x}{2}$

B. $\frac{2x - 1}{2}$

C. $x^2 - x$

D. $x - 1$

(1 mark)

9 $f(x) = 3x$

Choose the expression for $f^{-1}(x)$

A. $-3x$

B. $\frac{3}{x}$

C. $\frac{1}{3x}$

D. $\frac{x}{3}$

(1 mark)

10 (a) A function is represented by the following function machine.



A number is input into the machine.

The output is used as a new input.

The second output is 11.

Work out the number that was the **first input**.

(2 marks)

(b) A number is input into the machine.

The output given is the same number.

Work out the number.

(3 marks)

Hard Questions

1 $f(x) = 3x^2 - 2x - 8$

Express $f(x + 2)$ in the form $ax^2 + bx$

(3 marks)

2 (a) The functions f and g are such that $f(x) = x + 3$ and $g(x) = \frac{1}{x-2}$

Find $fg(x)$

Give your answer as a single algebraic fraction expressed as simply as possible.

(3 marks)

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \dots$

(3 marks)

3 (a) The function f is such that

$$f(x) = 4x - 1$$

Find $f^{-1}(x)$

(2 marks)

(b) The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that $fg(2) = 12$

work out the value of k

(2 marks)

4 (a) The functions f and g are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

Find $f^{-1}(x)$

(2 marks)

(b) Given that $fg(x) = 2gf(x)$,

show that $15x^2 - 12x - 1 = 0$

(5 marks)

5 $f(x) = 2x - 3$ and $g(x) = x^2$

Show that $f^{-1}(55) = fg(4)$

(4 marks)

6 $f(x) = \frac{x}{3} + 4$ for all values of x .

$g(x) = 6x^2 + 3$ for all values of x .

Work out $fg(x)$.

Give your answer in the form $ax^2 + b$ where a and b are integers.

(2 marks)

$$f(x) = cx + d$$

7 $f(4) = 7$

$$f(10) = 22$$

Work out the values of c and d .

$$c = \dots\dots\dots$$

$$d = \dots\dots\dots$$

(3 marks)

8 $f(x) = 3x^2 - 4x + 8$ for all values of x

Jenny says,

" $f(10)$ must equal $2 \times f(5)$, because 10 is 2×5 "

Is Jenny correct?

Show working to support your answer.

(2 marks)

9 $g(x) = 16 - x$ $h(x) = x^3$

Solve $gh(x) = 24$

(3 marks)

10 $f(x) = \frac{x}{x+2}$ $g(x) = x^2 - 2$

Work out $fg(x)$

Give your answer in the form $a + bx^n$ where a , b and n are integers.

(3 marks)

11 $f(x) = \frac{2x + 3}{x - 4}$

Work out $f^{-1}(x)$

(4 marks)

12 $g(x) = 3x + 7$

Solve $g^{-1}(x) = 2x$

$x = \dots\dots\dots$

(3 marks)

13 A function machine is shown below.



If the Input is 3, the Output is 5.

If the Input is 7, the Output is 25.

Use this information to fill in the two boxes.

(3 marks)

14 (a) Let

$$f(x) = (x - 1)^2$$

$$g(x) = 3x + 2$$

where $x > 0$.

Let

$$h(x) = fg(x)$$

Write an expression for $h(x)$ in the form $ax^2 + bx + c$.

(3 marks)

(b) Find $h^{-1}(x)$ when $x > 0$.

(3 marks)

Very Hard Questions

1 (a) Show that $\frac{x^2 + 3x}{2x^2 + 5x - 3}$ can be written as $\frac{x}{kx - 1}$

State the value of k .

(2 marks)

(b) $f(x) = \frac{x}{2x - 1}$

Find the inverse function f^{-1} in the form $f^{-1}(x) = \dots\dots$
Show your working clearly.

(3 marks)

2 The functions f and g are such that

$$f(x) = 5x + 3 \quad g(x) = ax + b \text{ where } a \text{ and } b \text{ are constants.}$$

$$g(3) = 20 \quad \text{and} \quad f^{-1}(33) = g(1)$$

Find the value of a and the value of b .

(5 marks)

3 $f(x) = \frac{1}{2}x$ $g(x) = x - x^2$

Solve $f^{-1}(x) = gf(x)$

(4 marks)

4 $f(x) = \frac{2x}{5} - 1$

Work out the value of $f^{-1}(3) + f(-0.5)$

(5 marks)

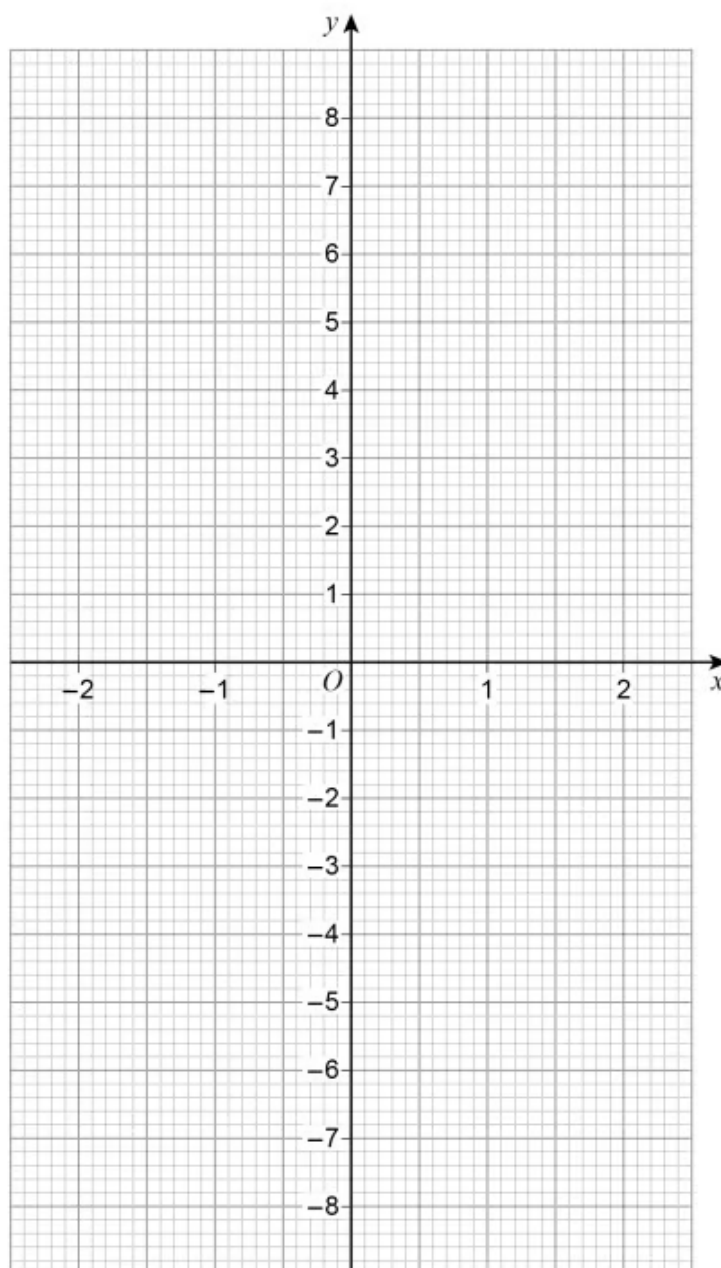
5 $f(x) = 5 - x$ and $g(x) = 3x + 7$

Simplify $f(2x) + g(x - 1)$

(3 marks)

6 $h(x) = \sqrt[3]{x}$ for all values of x

On the grid, draw the graph of the inverse function $y = h^{-1}(x)$ for $-2 \leq x \leq 2$



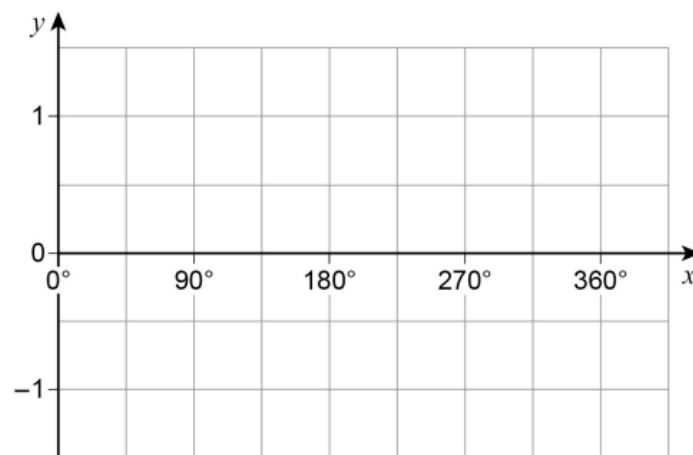
(2 marks)

7 For all values of x

$$f(x) = \sin x$$

$$g(x) = x + 90$$

On the grid, draw the graph of the composite function $y = fg(x)$ for $0^\circ \leq x \leq 360^\circ$



(2 marks)

8 $f(x) = 2x + c$

$$g(x) = cx + 5$$

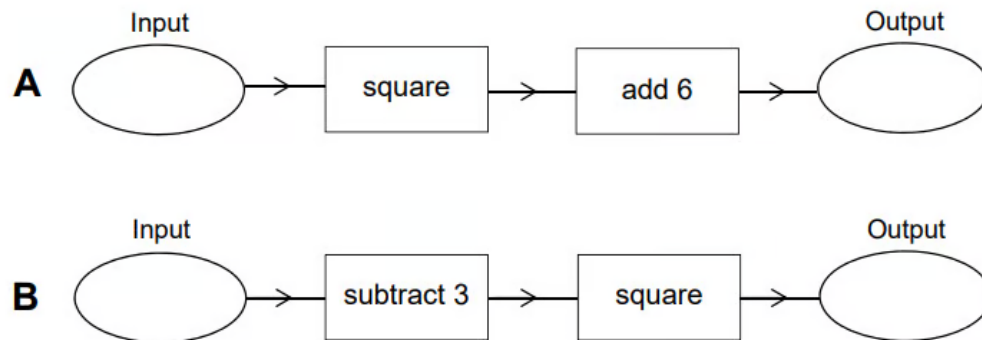
$$fg(x) = 6x + d$$

c and d are constants.

Work out the value of d .

(3 marks)

9 Here are two function machines, **A** and **B**.



Both machines have the same input.

Work out the range of input values for which the output of **A** is **less** than the output of **B**.

(4 marks)