

Iteration

Iteration

Easy (5 questions)	/16
Medium (8 questions)	/34
Hard (6 questions)	/26
Total Marks	/76

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Easy Questions

1 (a) Using $x_{n+1} = -2 - \frac{4}{x_n^2}$

with $x_0 = -2.5$

find the values of x_1 , x_2 and x_3

(3 marks)

(b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

(2 marks)

2 A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{4}{u_n - 1} \quad u_1 = 9$$

Work out the values of u_2 and u_3

(2 marks)

3 (a) An approximate solution to an equation is found using the iterative formula

$$x_{n+1} = \frac{(x_n)^3 - 2}{10} \quad \text{with } x_1 = -1$$

Work out the values of x_2 and x_3

$$x_2 = \dots\dots\dots$$

$$x_3 = \dots\dots\dots$$

(2 marks)

(b) Work out the solution to 5 decimal places.

$$x = \dots\dots\dots$$

(1 mark)

4 (a) An approximate solution to an equation is found using this iterative process.

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \text{ and } x_1 = -1$$

Work out the values of x_2 and x_3

$$x_2 = \dots\dots\dots$$

$$x_3 = \dots\dots\dots$$

(2 marks)

(b) Work out the solution to 6 decimal places.

$$x = \dots\dots\dots$$

(1 mark)

5 Use the formula $x_{n+1} = \frac{(x_n)^3}{30} + 2$ with $x_1 = 2$ to calculate x_2 and x_3 .

Round your answers correct to 4 decimal places.

$$x_2 = \dots\dots\dots \text{ and } x_3 = \dots\dots\dots$$

(3 marks)

Medium Questions

1 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

(2 marks)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7-x}$

(1 mark)

(c) Starting with $x_0 = 2$,

use the iteration formula $x_{n+1} = \sqrt[3]{7-x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

(3 marks)

2 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

(2 marks)

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

(1 mark)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

(3 marks)

3 (a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

(2 marks)

(b) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \text{ with } x_0 = 3.2,$$

find the values of x_1 , x_2 and x_3

(3 marks)

(c) Explain what the values of x_1 , x_2 , and x_3 represent.

(1 mark)

4 (a) A sphere has radius r cm

An approximate value of r can be found using the iterative formula

$$r_{n+1} = \sqrt{\frac{239}{r_n}}$$

The starting value is $r_1 = 7$

Work out the values of r_2 and r_3

$$r_1 = \dots\dots\dots$$

$$r_2 = \dots\dots\dots$$

(2 marks)

(b) Continue the iteration to work out the radius to 1 decimal place.

..... cm

(1 mark)

5 $x_{n+1} = \sqrt[3]{3x_n + 7}$

Use a starting value of $x_1 = 2$ to work out a solution to $x = \sqrt[3]{3x + 7}$

Give your answer to 3 decimal places.

(3 marks)

6 (a) At the start of the year n , the number of trees in a forest is T_n .

At the start of the following year, the number of trees is given by

$$T_{n+1} = kT_n$$

where k is a positive constant.

The number of trees at the start of the first year is 10 000.

The number of trees at the start of the second year is 8000.

Work out the number of trees at the start of the third year.

(3 marks)

- (b)** At the start of the fifth year, the value of k is increased by 0.25 to a new constant value.

Juwariah thinks that from the start of the fifth year, the number of trees in the forest will increase each year.

Is Juwariah correct?

You must give a reason for your answer.

(1 mark)

7 (a) The value of a second-hand watch at the start of the year n is V_n .

The value of the watch at the start of the year $n + 1$ is given by the formula $V_{n+1} = rV_n$

Given that r has a constant value of 0.85, find how many years it takes for the value of the watch to fall to half of its original value.

You must show your working clearly.

(2 marks)

(b) Sofia says the value of the watch may start to increase again in the future.

Give one reason why this might happen.

(1 mark)

8 (a) The value of a second-hand watch at the start of the year n is V_n .

The value of the watch at the start of the year $n + 1$ is given by the formula $V_{n+1} = rV_n$

Given that r has a constant value of 0.85, find how many years it takes for the value of the watch to fall to half of its original value.

You must show your working clearly.

(2 marks)

(b) Sofia says the value of the watch may start to increase again in the future.

Give one reason why this might happen.

(1 mark)

Hard Questions

1 (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

(2 marks)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2 + 7}$

(2 marks)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{5}{x_n^2 + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

(3 marks)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$, comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

(2 marks)

2 The number of bees in a beehive at the start of year n is P_n .

The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

(3 marks)

- 3** The number of slugs in a garden t days from now is p_t where

$$p_0 = 100$$

$$p_{t+1} = 1.06p_t$$

Work out the number of slugs in the garden 3 days from now.

(3 marks)

- 4 (a)** The number of rabbits on a farm at the end of month n is P_n

The number of rabbits at the end of the next month is given by $P_{n+1} = 1.2 P_n - 50$

At the end of March there are 200 rabbits on the farm.

Work out how many rabbits there will be on the farm at the end of June.

(3 marks)

- (b)** Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time.

(1 mark)

- 5 At the start of year n , the number of animals in a population is P_n . At the start of the following year, the number of animals in the population is P_{n+1} , where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000

At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k .

(3 marks)

- 6 (a) Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1 , x_2 and x_3
Start with $x_0 = 2$

$$x_1 = \dots\dots$$

$$x_2 = \dots\dots$$

$$x_3 = \dots\dots$$

(3 marks)

- (b) The values of x_1 , x_2 and x_3 found part (a) are estimates of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

Find the value of a and the value of b .

$$a = \dots\dots$$

$$b = \dots\dots$$

(1 mark)