**GCSE · Edexcel · Maths**

3 hours 50 questions

Exam Questions

# Vectors

Introduction to Column Vectors / Representing Vectors as Diagrams / Length of a Vector / Position & Displacement Vectors / Finding Vector Paths / Problem Solving with Vectors

| | |
|---|---|
| Medium (16 questions) | /46 |
| Hard (19 questions) | /78 |
| Very Hard (15 questions) | /69 |
| **Total Marks** | **/193** |

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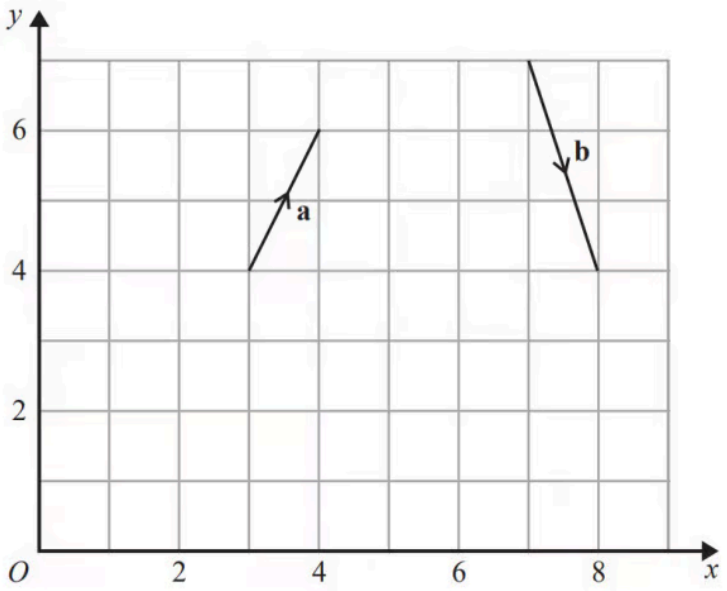
# Medium Questions

**1 (a)** The vector $\mathbf{a}$ and the vector $\mathbf{b}$ are shown on the grid.




On the grid, draw and label vector $-2\mathbf{a}$

**(1 mark)**

**(b)** Work out $\mathbf{a} + 2\mathbf{b}$ as a column vector.

**(2 marks)**

**2 (a)**

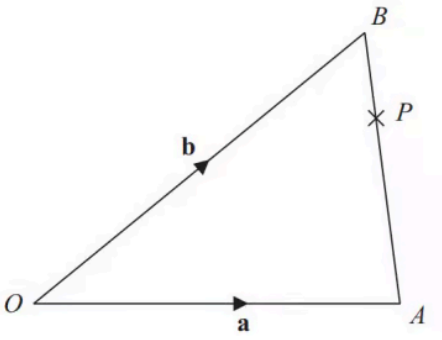



Diagram **NOT** accurately drawn

$OAB$ is a triangle.

$\overrightarrow{OA} = \mathbf{a}$

$\overrightarrow{OB} = \mathbf{b}$

Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

**(1 mark)**

**(b)** $P$ is the point on $AB$ such that $AP : PB = 3 : 1$

Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

**(3 marks)**

**3 (a)**

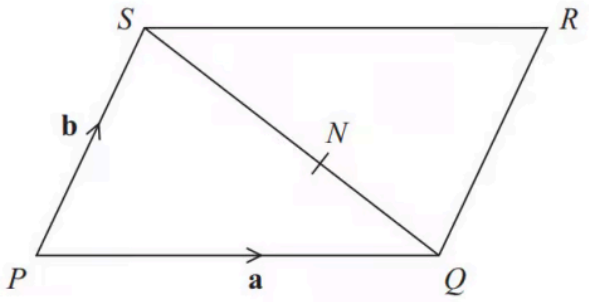



Diagram **NOT** accurately drawn

$PQRS$ is a parallelogram.
$N$ is the point on $SQ$ such that $SN : NQ = 3:2$

$\overrightarrow{PQ} = \mathbf{a}$
$\overrightarrow{PS} = \mathbf{b}$

Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, an expression for $\overrightarrow{SQ}$.

**(1 mark)**

**(b)** Express $\overrightarrow{NR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

**(3 marks)**

**4**

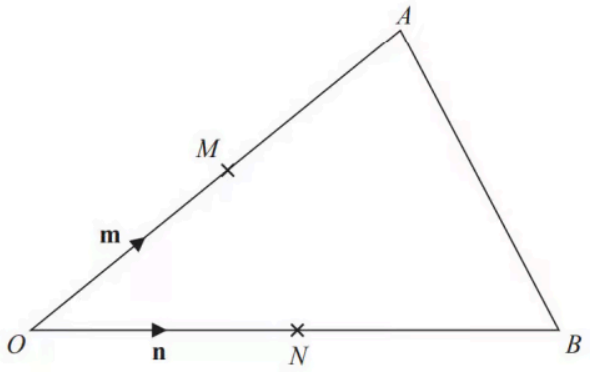



Diagram **NOT** accurately drawn

$OAB$ is a triangle.
$M$ is the midpoint of $OA$.
$N$ is the midpoint of $OB$.

$\overrightarrow{OM} = \mathbf{m}$
$\overrightarrow{ON} = \mathbf{n}$

Show that $AB$ is parallel to $MN$.

**(3 marks)**

**5 (a)**

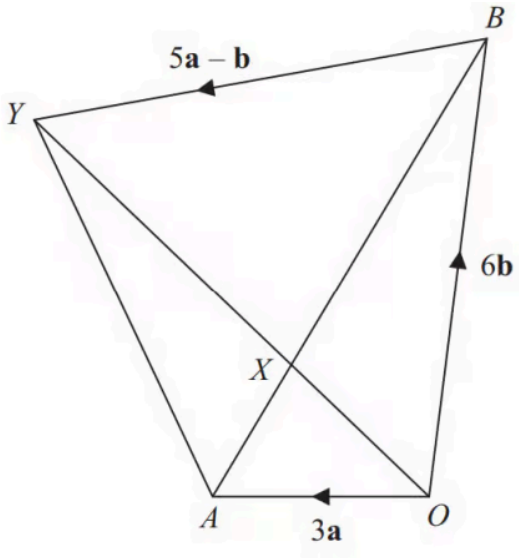



Diagram **NOT** accurately drawn

$OAYB$ is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$

$\overrightarrow{OB} = 6\mathbf{b}$

Express $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

**(1 mark)**

**(b)** $X$ is the point on $AB$ such that $AX : XB = 1 : 2$ and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$

**(4 marks)**

**6** Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Find, as a column vector, $\overrightarrow{AC}$

**(2 marks)**

**7** $ABCDEF$ and $GHIJKL$ are regular hexagons each with centre $O$.

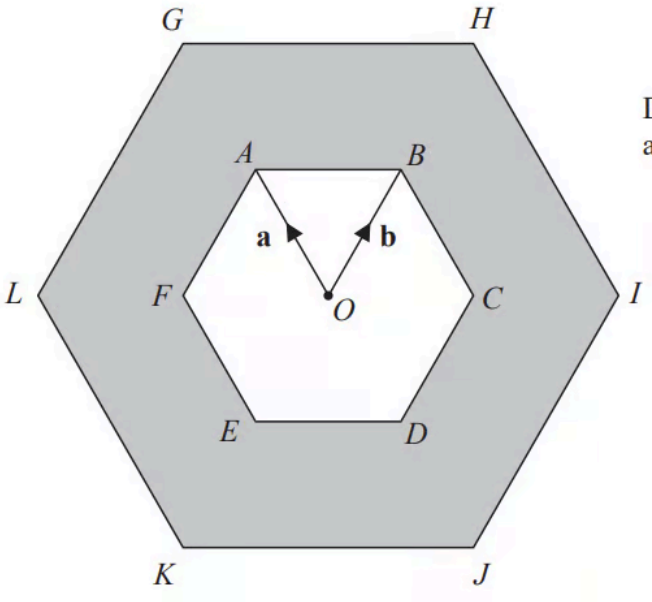



Diagram **NOT** accurately drawn

$GHIJKL$ is an enlargement of $ABCDEF$, with centre $O$ and scale factor 2

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

Write the following vectors, in terms of $\mathbf{a}$ and $\mathbf{b}$.
Simplify your answers.

i) $\overrightarrow{AB}$

[1]

ii) $\overrightarrow{KI}$

[2]

iii) $\overrightarrow{LD}$

[2]

**(1 mark)**

**8** $ABCDEF$ is a regular hexagon.

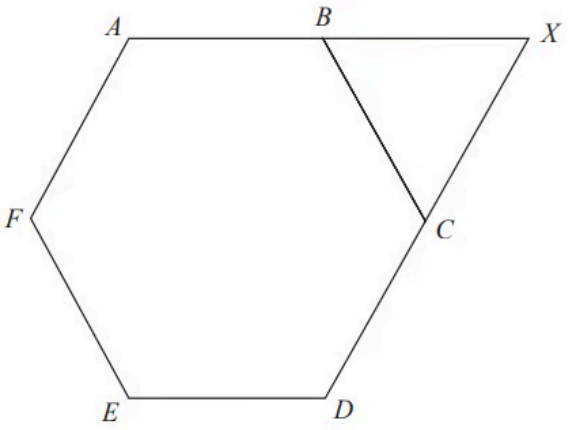



Diagram **NOT** accurately drawn

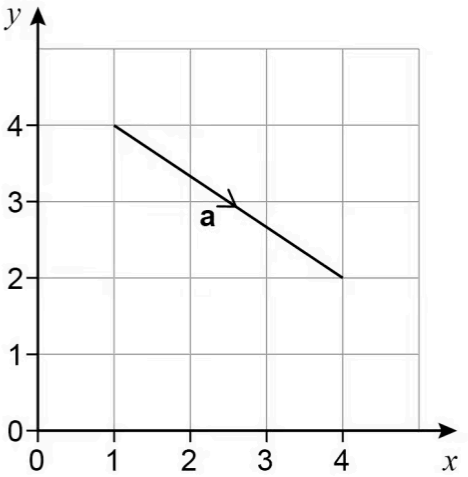
$ABX$ and $DCX$ are straight lines.

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$

Find $\overrightarrow{EX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

**(4 marks)**

**9** Here is vector $\mathbf{a}$.




Choose the column vector that represents $\mathbf{a}$.

**A.** $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

**B.** $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

**C.** $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

**D.** $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

**(1 mark)**

**10** $\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

Work out $\mathbf{a} - 3\mathbf{b}$

**A.** $\begin{pmatrix} -6 \\ 17 \end{pmatrix}$

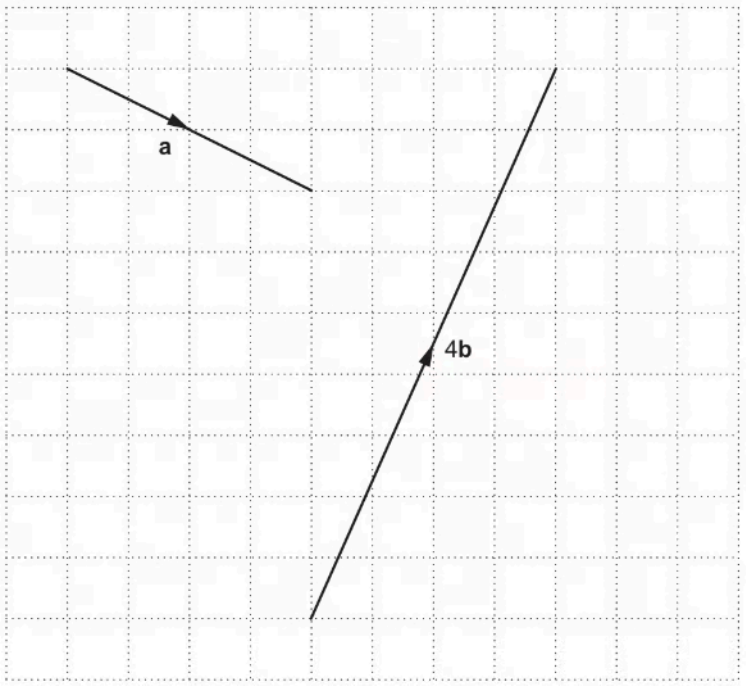
**B.** $\begin{pmatrix} -6 \\ -13 \end{pmatrix}$

**C.** $\begin{pmatrix} 0 \\ 17 \end{pmatrix}$

**D.** $\begin{pmatrix} 0 \\ -13 \end{pmatrix}$

**(1 mark)**

**11 (a)** Vectors $\mathbf{a}$ and $4\mathbf{b}$ are drawn on the grid.




Write vector $\mathbf{a}$ as a column vector.

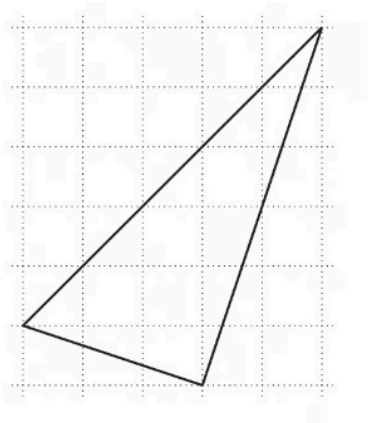
**(2 marks)**

**(b)** Find vector $\mathbf{b}$ as a column vector.

**(2 marks)**

**12** Vector $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and vector $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Gavin starts to draw a diagram to show that $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$



Complete Gavin's diagram.

**(3 marks)**

**13** $\mathbf{b}$ is a vector.

Given that $\mathbf{b} + \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ is parallel to $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, find two possible answers for $\mathbf{b}$.

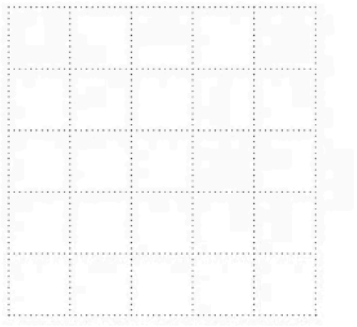
$\mathbf{b} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ or $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$

**(3 marks)**

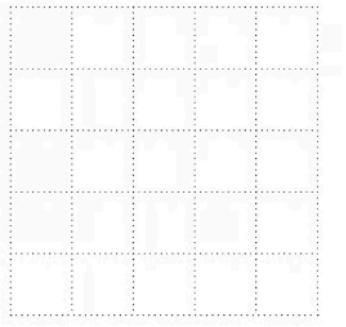
**14 (a)** Vector $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, vector $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

On each grid below, draw a vector to represent

(i) $2\mathbf{a}$,



(ii) $\mathbf{a} + \mathbf{b}$.



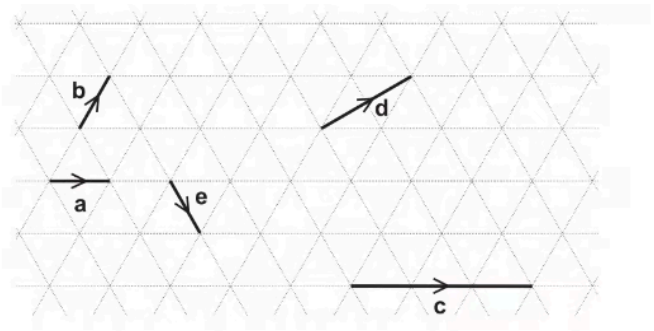
**(2 marks)**

**(b)** Emma says that if she draws vector $\mathbf{a}$ and vector $\mathbf{b}$ they will be the same.

Explain why this is incorrect.

**(1 mark)**

**15** Vectors **a**, **b**, **c**, **d** and **e** are drawn on an isometric grid.




Write each of the vectors **c**, **d** and **e** in terms of **a** and/or **b**.

**c** = .....

**d** = .....

**e** = .....

**(3 marks)**

16 $\mathbf{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \mathbf{b} = \begin{pmatrix} p \\ 5 \end{pmatrix}$

$\mathbf{a}$ and $\mathbf{b}$ are column vectors.

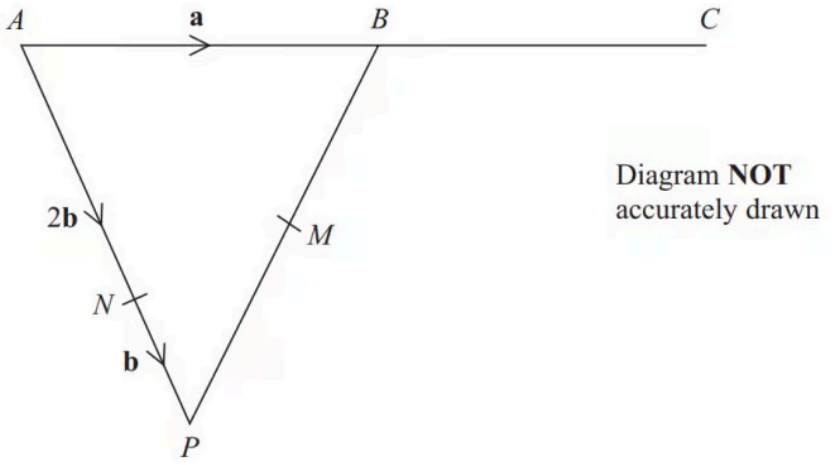
$$2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$$

Find the value of $p$.

**(2 marks)**

# Hard Questions

**1 (a)**




$APB$ is a triangle.
$N$ is a point on $AP$.

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{AN} = 2\mathbf{b}$ $\overrightarrow{NP} = \mathbf{b}$

Find the vector $\overrightarrow{PB}$, in terms of $\mathbf{a}$ and $\mathbf{b}$.

**(1 mark)**

**(b)** $B$ is the midpoint of $AC$.
$M$ is the midpoint of $PB$.

Show that $NMC$ is a straight line.

**(4 marks)**

**2 (a)**

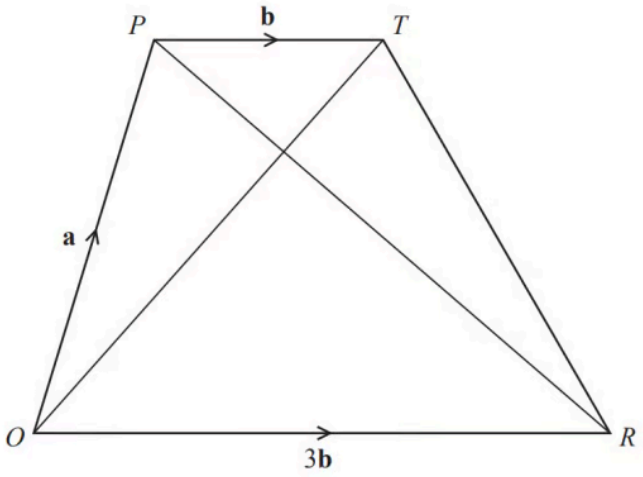



Diagram **NOT** accurately drawn

$OPTR$ is a trapezium.

$\overrightarrow{OP} = \mathbf{a}$

$\overrightarrow{PT} = \mathbf{b}$

$\overrightarrow{OR} = 3\mathbf{b}$

i) Find $\overrightarrow{OT}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

[1]

ii) Find $\overrightarrow{PR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ Give your answer in its simplest form.

[1]

**(2 marks)**

**(b)** $S$ is the point on $PR$ such that $PS : SR = 1 : 3$

Find $\overrightarrow{OS}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

**(2 marks)**

**(c)** What does your answer to part (b) tell you about the position of point $S$?

**(2 marks)**

**3**

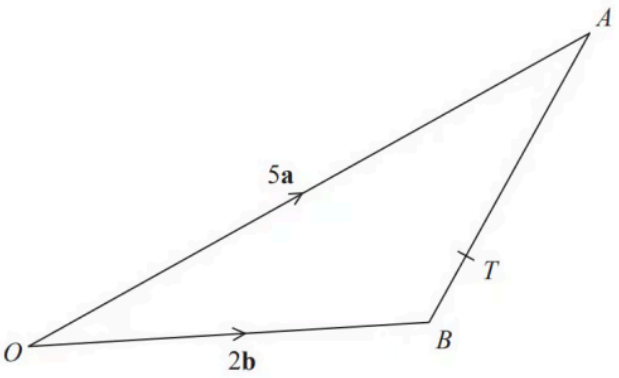



Diagram **NOT** accurately drawn

$OAB$ is a triangle.

$\overrightarrow{OA} = 5\mathbf{a}$

$\overrightarrow{OB} = 2\mathbf{b}$

$T$ is the point on $AB$ such that $AT : TB = 5 : 1$

Show that $OT$ is parallel to the vector $\mathbf{a} + 2\mathbf{b}$

**(4 marks)**

**4 (a)**

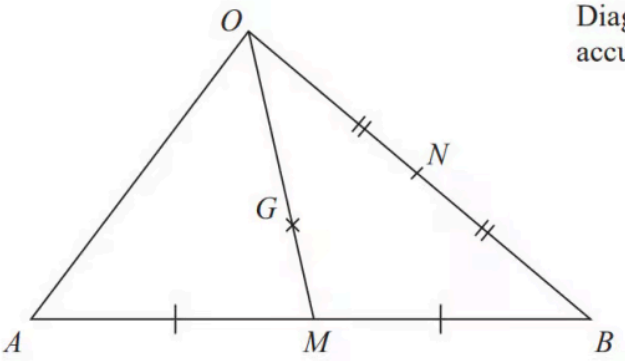



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 6\mathbf{b}$
$M$ is the midpoint of $AB$.

Write $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

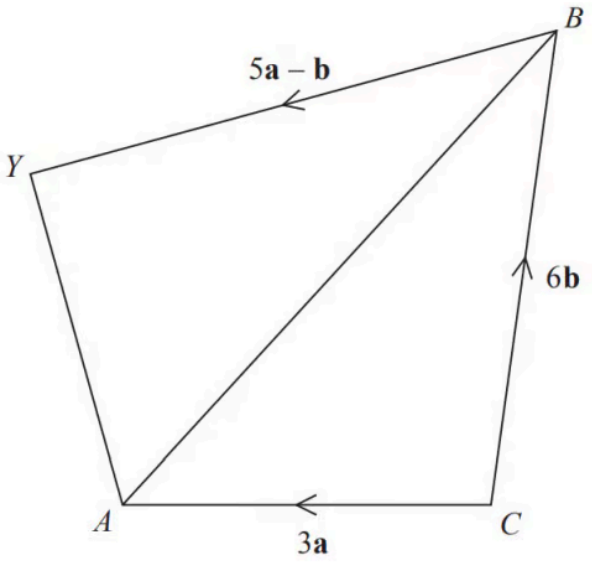
**(2 marks)**

**(b)** $N$ is the midpoint of $OB$.
$G$ is the point on $OM$ such that $OG : GM = 2 : 1$

Show that $AGN$ is a straight line.

**(4 marks)**

**5**




$CAYB$ is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$

$\overrightarrow{CB} = 6\mathbf{b}$

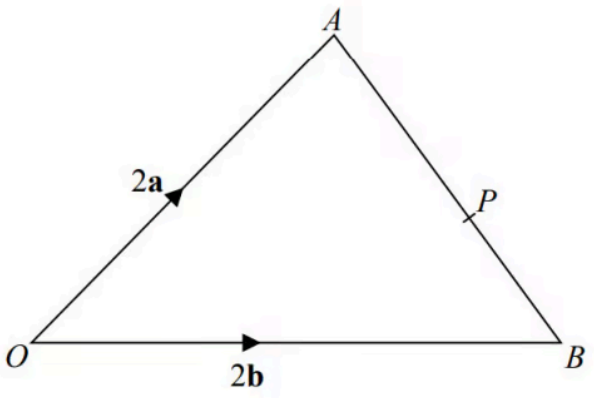
$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

$X$ is the point on $AB$ such that $AX:XB = 1:2$

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

**(5 marks)**

6




$OAB$ is a triangle.
$P$ is the point on $AB$ such that $AP : PB = 5:3$

$\overrightarrow{OA} = 2\mathbf{a}$
$\overrightarrow{OB} = 2\mathbf{b}$

$\overrightarrow{OP} = k(3\mathbf{a} + 5\mathbf{b})$ where $k$ is a scalar quantity.

Find the value of $k$.

**(4 marks)**

**7**

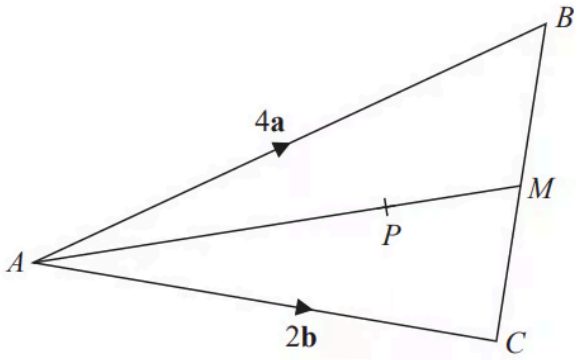



Diagram **NOT** accurately drawn

$ABC$ is a triangle.
The midpoint of $BC$ is $M$.
$P$ is a point on $AM$.

$\overrightarrow{AB} = 4\mathbf{a}$

$\overrightarrow{AC} = 2\mathbf{b}$

$\overrightarrow{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$

Find the ratio $AP : PM$

**(3 marks)**

**8** $OAB$ is a triangle.

$\overrightarrow{OA} = \mathbf{a}$

$\overrightarrow{OB} = \mathbf{b}$

The point $C$ lies on $OA$ such that $OC : CA = 1 : 2$
The point $D$ lies on $OB$ such that $OD : DB = 1 : 2$

Using a vector method, prove that $ABDC$ is a trapezium.

**(3 marks)**

**9 (a)** The diagram shows parallelogram $ABCD$.

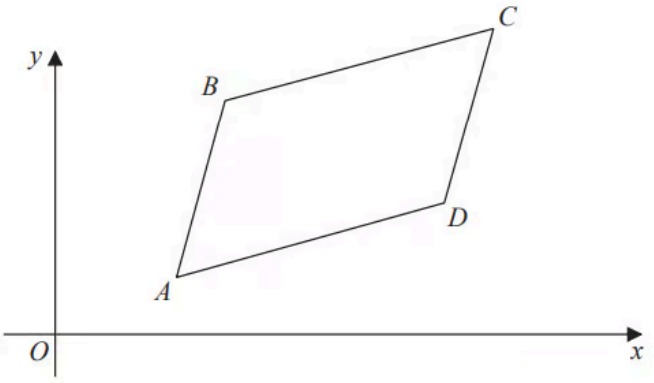



Diagram **NOT** accurately drawn

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \quad \overrightarrow{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$$

The point $B$ has coordinates (5, 8)
Work out the coordinates of the point $C$.

**(3 marks)**

**(b)** The point $E$ has coordinates (63, 211)
Use a vector method to prove that $ABE$ is a straight line.

**(2 marks)**

**10 (a)** The diagram shows triangle $ABD$.

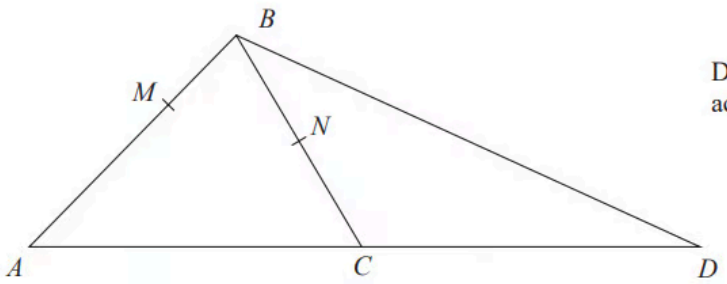



Diagram **NOT** accurately drawn

$N$ is the midpoint of $BC$.

$C$ is the midpoint of $AD$.

$M$ is the point on $AB$ such that $AM:MB = 3:1$

$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$

Express, in terms of $\mathbf{p}$ and $\mathbf{q}$,

i) $\overrightarrow{BD}$

ii) $\overrightarrow{MN}$

**(3 marks)**

**(b)** State, giving reasons, two different geometric facts relating $MN$ and $BD$.

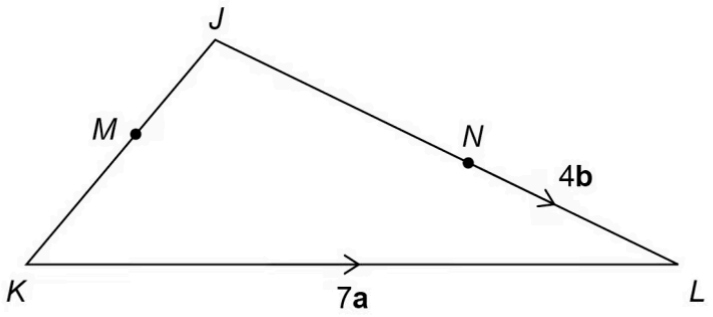
**(2 marks)**

**11** In triangle $JKL$

$M$ is the midpoint of
$JK$ $JN:NL = 3:2$

$\overrightarrow{KL} = 7\mathbf{a}$ $\overrightarrow{NL} = 4\mathbf{b}$

Not drawn accurately




Work out $\overrightarrow{JM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

**(3 marks)**

**12** $PQR$ is a straight line.

$PQ : QR = 3 : 1$

$\overrightarrow{PQ} = \mathbf{a}$

Not drawn accurately


Choose the vector $\overrightarrow{RQ}$

**A.** $\frac{1}{3}\mathbf{a}$

**B.** $\frac{1}{4}\mathbf{a}$

**C.** $-\frac{1}{3}\mathbf{a}$

**D.** $-\frac{1}{4}\mathbf{a}$

**(1 mark)**

**13 (a)** $\mathbf{a} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$

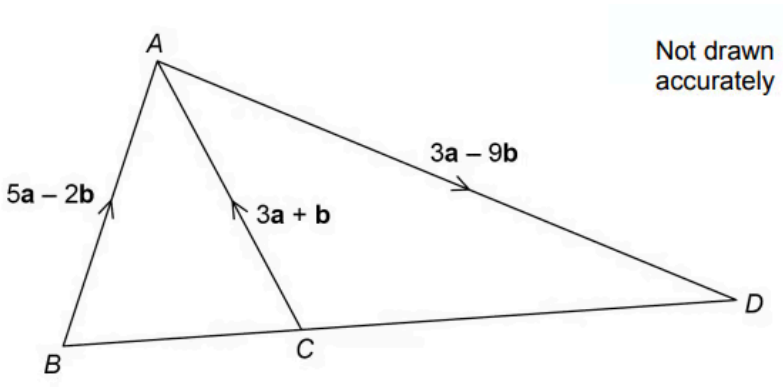
Work out $\mathbf{a} + \mathbf{b} + \mathbf{c}$

**(2 marks)**

**(b)** Show that $\mathbf{a} + 2\mathbf{c}$ is parallel to $\mathbf{b}$

**(2 marks)**

**14**




Is $BCD$ a straight line?
Show working to support your answer.

**(3 marks)**

**15** Given that

$$m\begin{pmatrix}4\\1\end{pmatrix} + n\begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}12\\6\end{pmatrix}$$

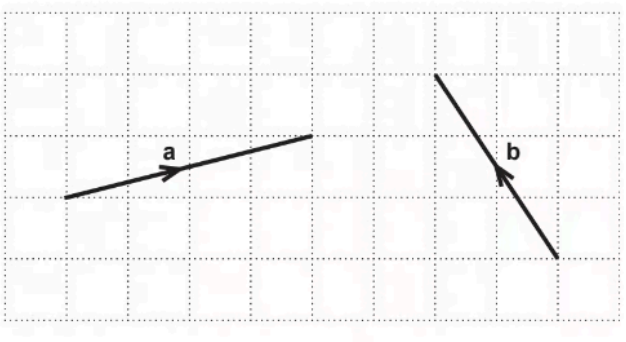
find the value of $m$ and the value of $n$.

$m$ = .....

$n$ = .....

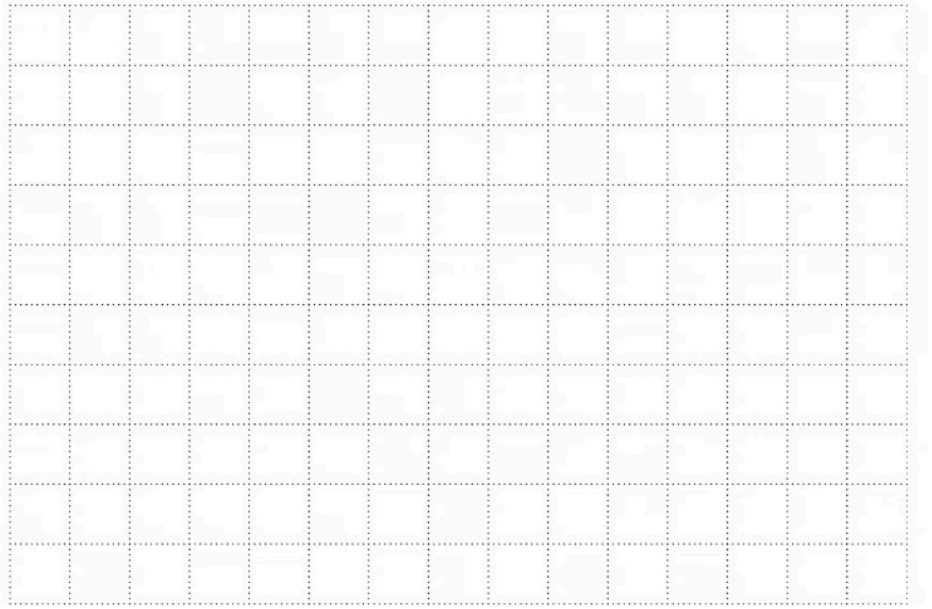
**(5 marks)**

**16** Two vectors, **a** and **b**, are shown on the 1 centimetre grid below.




Show that the vector **a** + 2**b** has length 7cm.

You may use the grid below.



**(3 marks)**

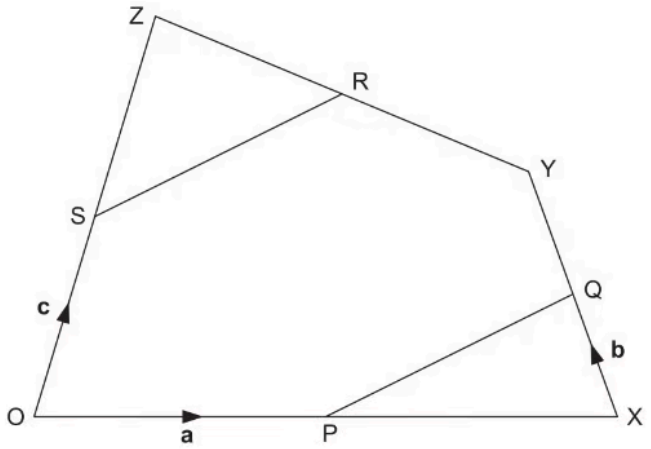
**17** Vector $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, vector $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and vector $\mathbf{c} = \begin{pmatrix} -12 \\ 0 \end{pmatrix}$

Find the value $k$ so that $k(\mathbf{a} - \mathbf{b}) = \mathbf{c}$

$k$ = .....

**(2 marks)**

**18** P, Q, R and S are the midpoints of OX, XY, YZ and OZ respectively.

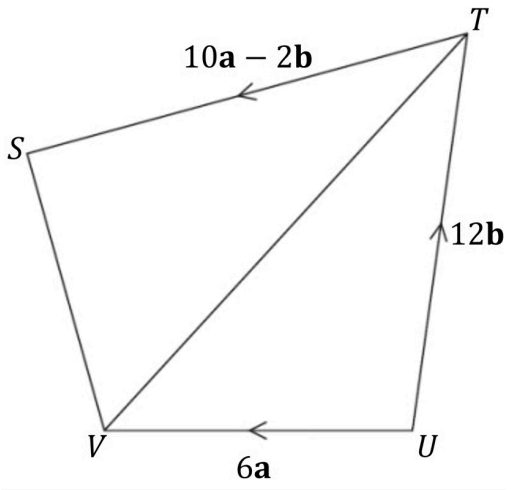



$\overrightarrow{OP} = \mathbf{a}$, $\overrightarrow{XQ} = \mathbf{b}$ and $\overrightarrow{OS} = \mathbf{c}$.

Show that $PQ$ is parallel to $SR$.

**(5 marks)**

**19 (a)** $STUV$ is a quadrilateral.




$\overrightarrow{TS} = 10\mathbf{a} - 2\mathbf{b}$

$\overrightarrow{UT} = 12\mathbf{b}$

$\overrightarrow{UV} = 6\mathbf{a}$

$W$ is the point on $VT$ such that $VW : WT = 1:2$.

Prove that $\overrightarrow{UW} = \frac{2}{3}\overrightarrow{WS}$

You must show all your working.

**(4 marks)**

**(b)** State two different facts about the geometric relationship between the point $U$, $W$ and $S$.

**(2 marks)**

# Very Hard Questions

**1 (a)** $OACB$ is a parallelogram.

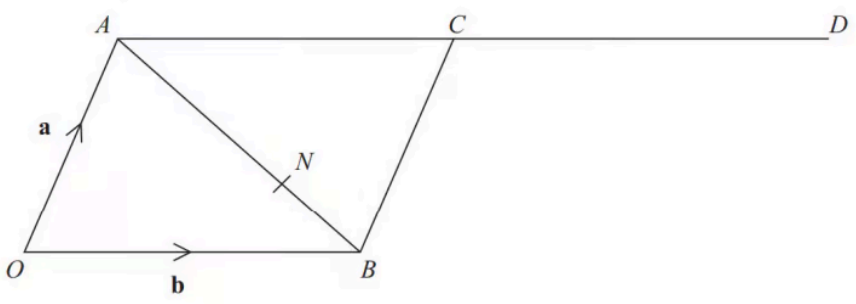



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

$D$ is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$
The point $N$ divides $AB$ in the ratio $2:1$

Write an expression for $\overrightarrow{ON}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

**(3 marks)**

**(b)** Prove that $OND$ is a straight line.

**(3 marks)**

**2**

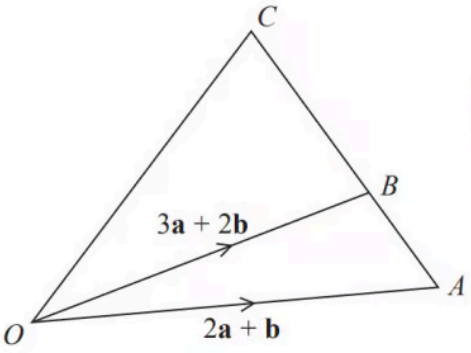



Diagram **NOT** accurately drawn

$ABC$ is a straight line.

$AB : BC = 2:5$

$\overrightarrow{OA} = 2\mathbf{a} + \mathbf{b}$

$\overrightarrow{OB} = 3\mathbf{a} + 2\mathbf{b}$

Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

**(4 marks)**

3

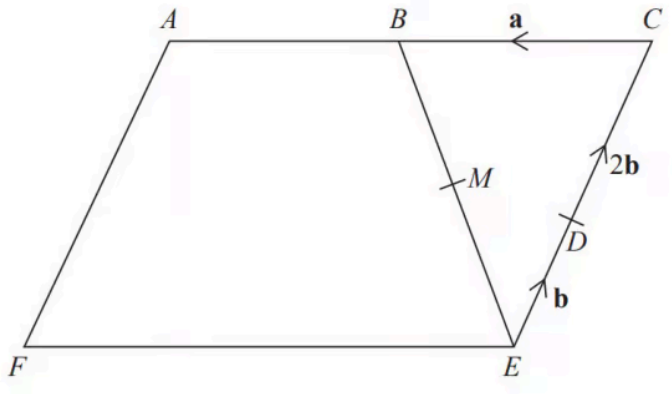



Diagram **NOT** accurately drawn

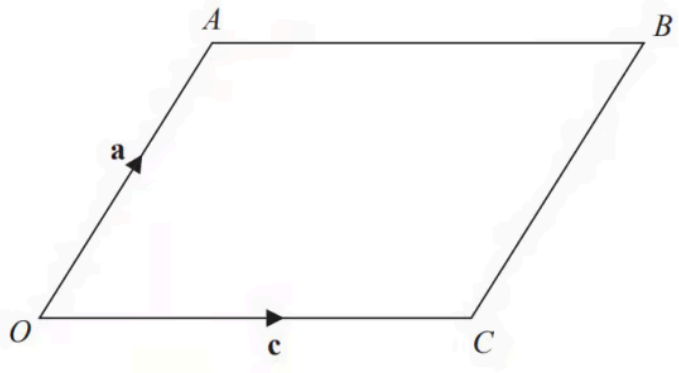
$ACEF$ is a parallelogram.
$B$ is the midpoint of $AC$.
$M$ is the midpoint of $BE$.

$\overrightarrow{CB} = \mathbf{a}$
$\overrightarrow{ED} = \mathbf{b}$
$\overrightarrow{DC} = \mathbf{2b}$

Show that $AMD$ is a straight line.

**(5 marks)**

**4**




$OABC$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

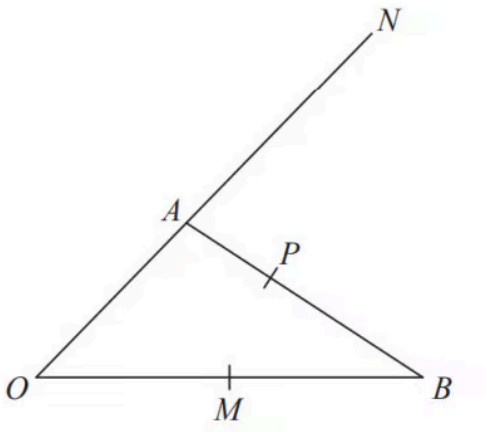
$X$ is the midpoint of the line $AC$.

$OCD$ is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$ find the value of $k$.

**(4 marks)**

5




$OAN$, $OMB$ and $APB$ are straight lines.
$AN = 2OA$.
$M$ is the midpoint of $OB$.

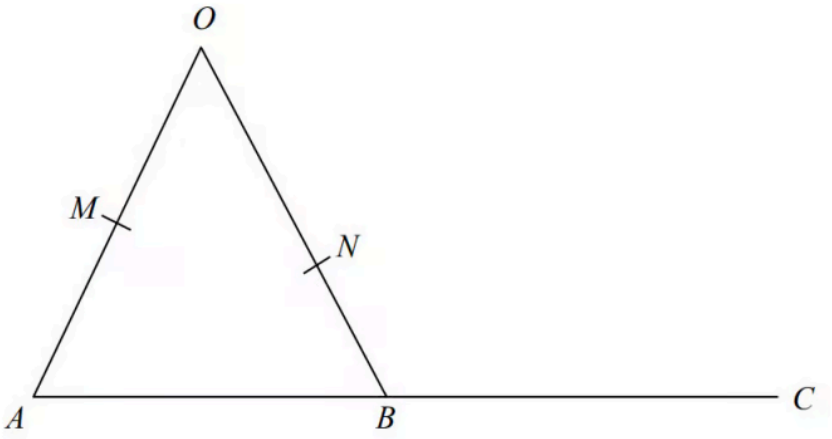
$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where $k$ is a scalar quantity.

Given that $MPN$ is a straight line, find the value of $k$.

**(5 marks)**

6



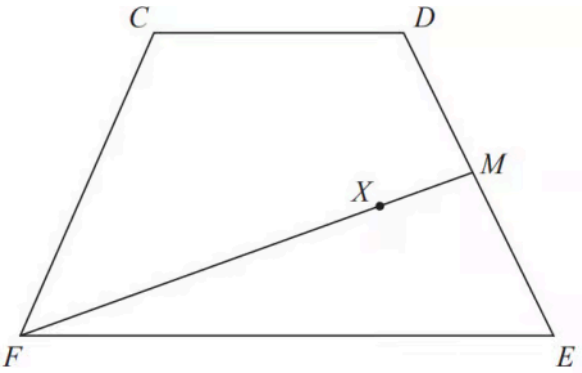

$OMA$, $ONB$ and $ABC$ are straight lines.
$M$ is the midpoint of $OA$.
$B$ is the midpoint of $AC$.

$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$
where $k$ is a scalar quantity.

Given that $MNC$ is a straight line, find the value of $k$.

**(5 marks)**

**7 (a)** $CDEF$ is a quadrilateral.




$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

Express $\overrightarrow{FE}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$.
Give your answer in its simplest form.

**(2 marks)**

**(b)** $M$ is the midpoint of $DE$.
$X$ is the point on $FM$ such that $FX:XM = n:1$
$CXE$ is a straight line.

Work out the value of $n$.

**(4 marks)**

**8** The diagram shows triangle $OAB$

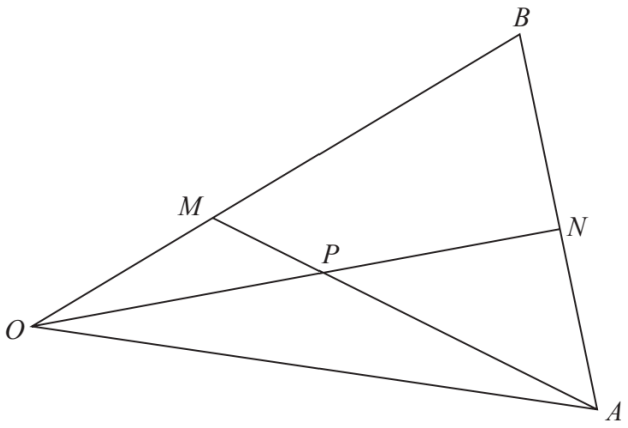



Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 8\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

$M$ is the point on $OB$ such that $OM : MB = 1 : 2$

$N$ is the midpoint of $AB$

$P$ is the point of intersection of $ON$ and $AM$

Using a vector method, find $\overrightarrow{OP}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

Show your working clearly.

$\overrightarrow{OP}$= .....

**(5 marks)**

**9** $OXYZ$ is a trapezium.

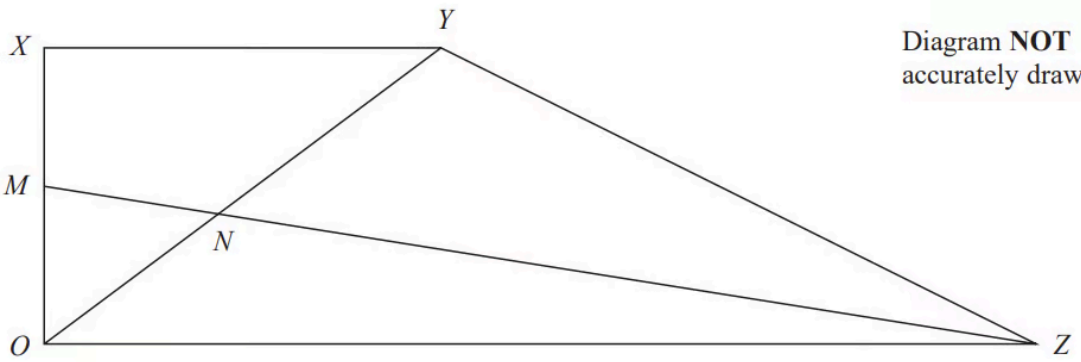



Diagram **NOT** accurately drawn

$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{XY} = \mathbf{b}$

$\overrightarrow{OZ} = 3\mathbf{b}$

$M$ is the midpoint of $OX$
$N$ is the point such that $MNZ$ and $ONY$ are straight lines.
Given that $ON : OY = \lambda : 1$
use a vector method to find the value of $\lambda$

$\lambda$ = .....

**(5 marks)**

**10 (a)** In the diagram

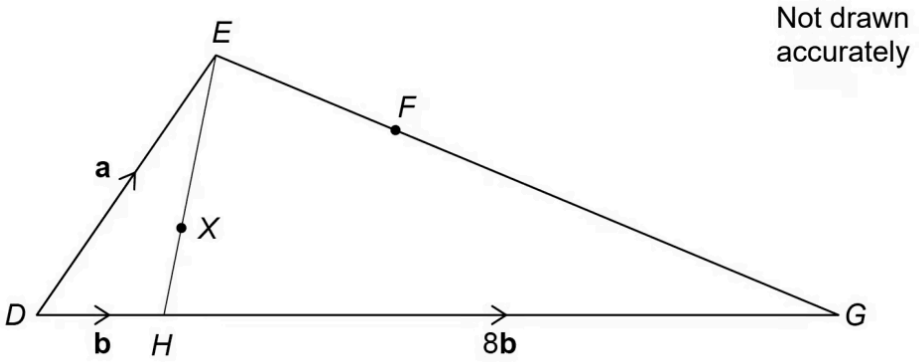
$\overrightarrow{DE} = \mathbf{a}$

$\overrightarrow{DH} = \mathbf{b}$

$\overrightarrow{HG} = 8\mathbf{b}$

$EX : XH = 3 : 1$

$EF : FG = 1 : 3$




Show that $\overrightarrow{DX} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

**(2 marks)**

**(b)** Is $DXF$ a straight line?

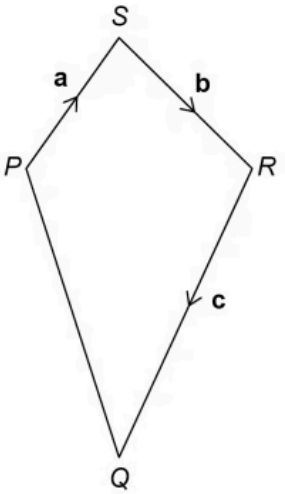
Show working to support your answer.

**(4 marks)**

**11** Here is quadrilateral $PQRS$.

$\overrightarrow{PS} = \mathbf{a}$ $\overrightarrow{SR} = \mathbf{b}$ $\overrightarrow{RQ} = \mathbf{c}$

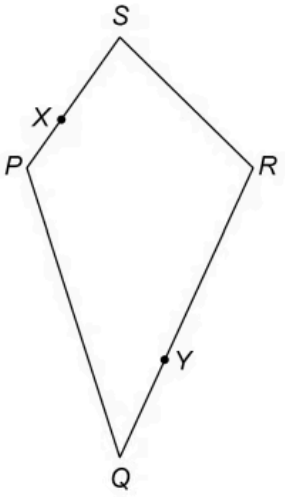
$\overrightarrow{PS}$ and $\overrightarrow{QR}$ are not parallel.




Not drawn accurately

$X$ is a point on $PS$ where $PX : XS = 1 : 2$

$Y$ is a point on $RQ$ where $RY : YQ = 2 : 1$



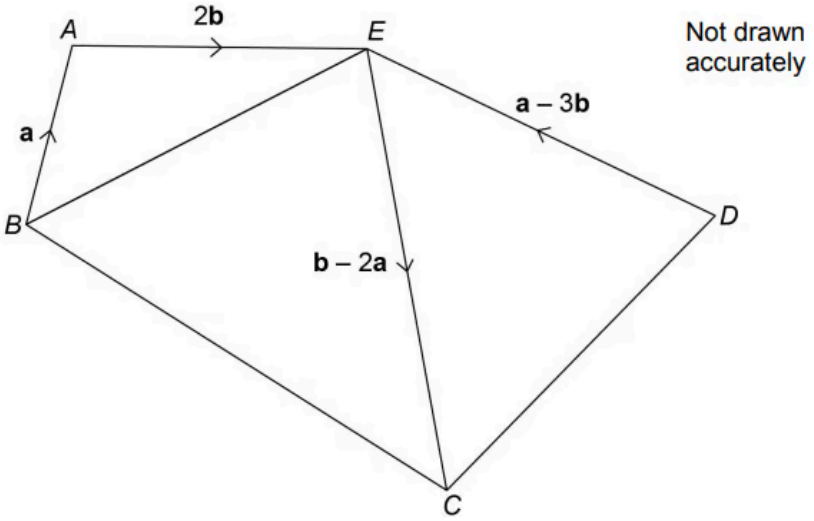

Not drawn accurately

Is $XY$ parallel to $PQ$?

Show working to support your answer.

**(3 marks)**

**12** $ABCDE$ is a pentagon.




Show that $BCDE$ is a parallelogram.

**(3 marks)**

**13** Vector **m**$=\begin{pmatrix} 2 \\ k \end{pmatrix}$ and vector **n** $=\begin{pmatrix} 3 \\ 11 \end{pmatrix}$

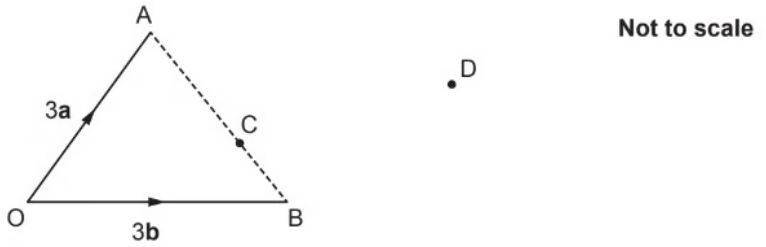
Vector 2**m** + **n** is parallel to $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Find the value of $k$.

$k =$ .....

**(4 marks)**

**14** The diagram shows triangle OAB and points C and D.




$\overrightarrow{OA} = 3\mathbf{a}$ and $\overrightarrow{OB} = 3\mathbf{b}$.

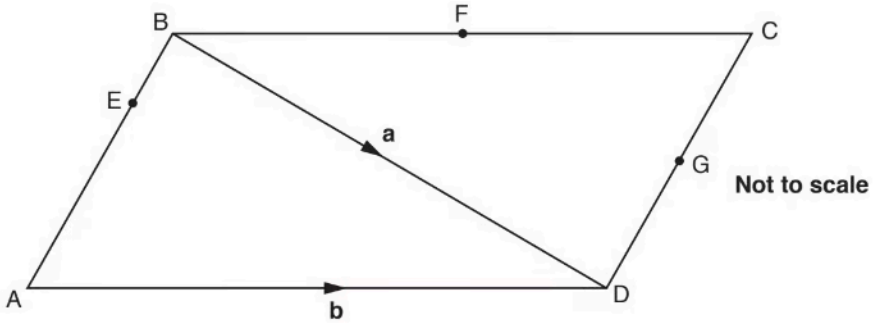
C lies on AB such that AC = 2CB.

D is such that $\overrightarrow{BD} = +2\mathbf{a} + \mathbf{b}$.

Show, using vectors, that OCD is a straight line.

**(5 marks)**

**15** ABCD is a parallelogram.




$\overrightarrow{BD} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.
F is the midpoint of BC.
G is the midpoint of DC.
$AE = 3EB$.

Prove that $\overrightarrow{EF}$ and $\overrightarrow{AG}$ are parallel.

**(3 marks)**