GCSE · Edexcel · Maths

3 hours 49 questions

Exam Questions

Congruence, Similarity & Geometrical Proof

Congruence / Congruent Triangles / Similarity / Similar Lengths / Geometrical Proof

Easy (14 questions)	/37
Medium (15 questions)	/49
Hard (10 questions)	/36
Very Hard (10 questions)	/46
Total Marks	**/168**

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Easy Questions

1 (a)

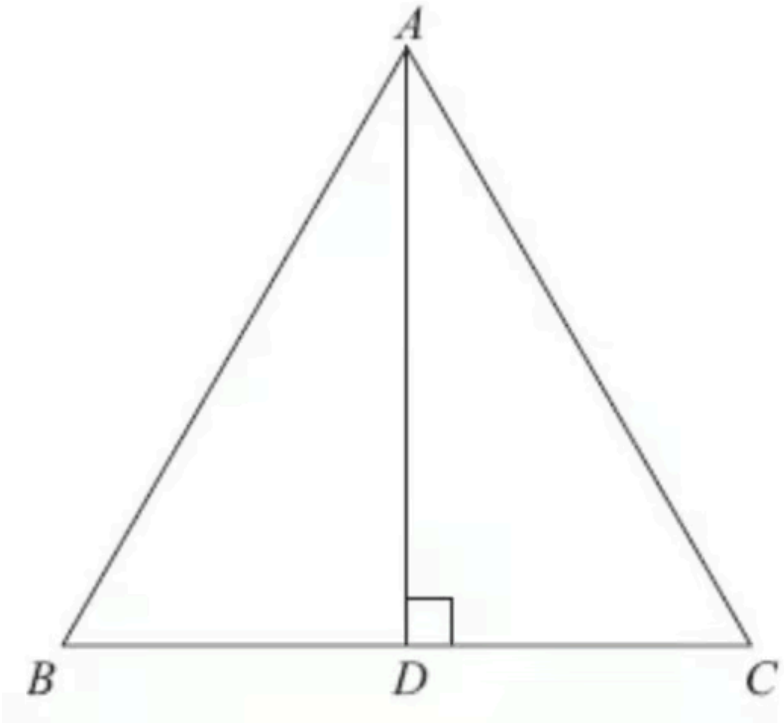


Diagram **NOT** accurately drawn

ABC is an equilateral triangle.
D lies on *BC*.
AD is perpendicular to *BC*.

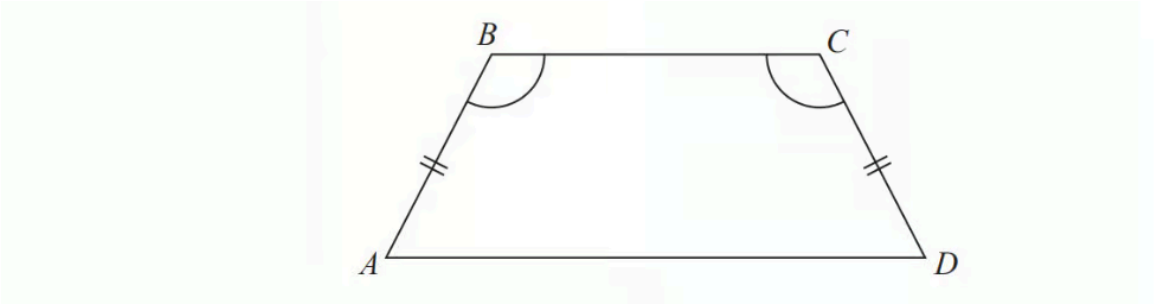
Prove that triangle *ADC* is congruent to triangle *ADB*.

(3 marks)

(b) Hence, prove that $BD = \frac{1}{2}AB$.

(2 marks)

2 $ABCD$ is a quadrilateral.

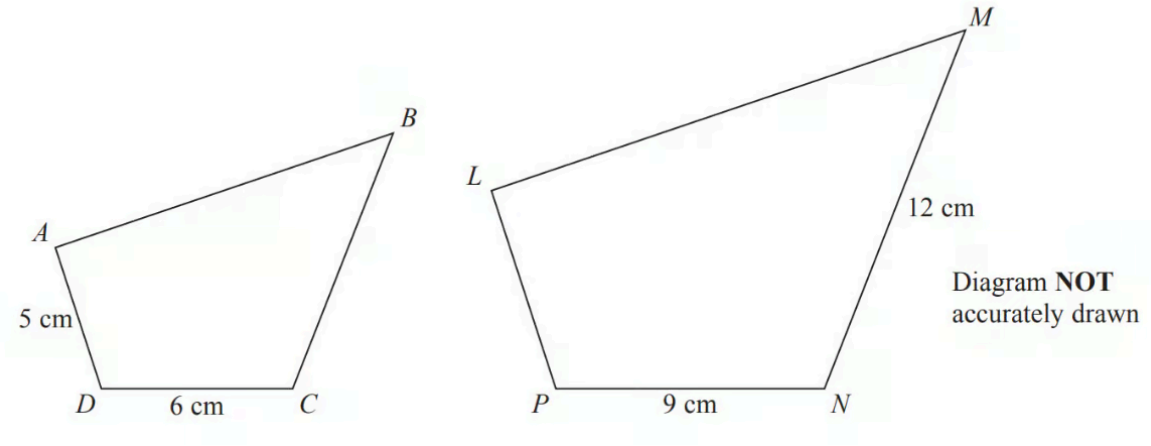


$AB = CD$.
Angle ABC = angle BCD.

Prove that $AC = BD$.

(4 marks)

3 (a)



Quadrilaterals *ABCD* and *LMNP* are mathematically similar.

Angle *A* = angle *L*
Angle *B* = angle *M*
Angle *C* = angle *N*
Angle *D* = angle *P*

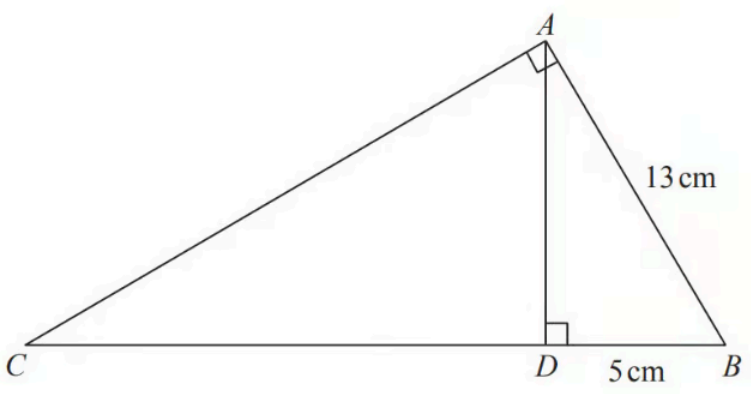
Work out the length of *LP*.

(2 marks)

(b) Work out the length of *BC*.

(2 marks)

4 ABC and ABD are two right-angled triangles.



Angle BAC = angle ADB = 90°

AB = 13 cm
DB = 5 cm

Work out the length of CB.

(3 marks)

5 (a)

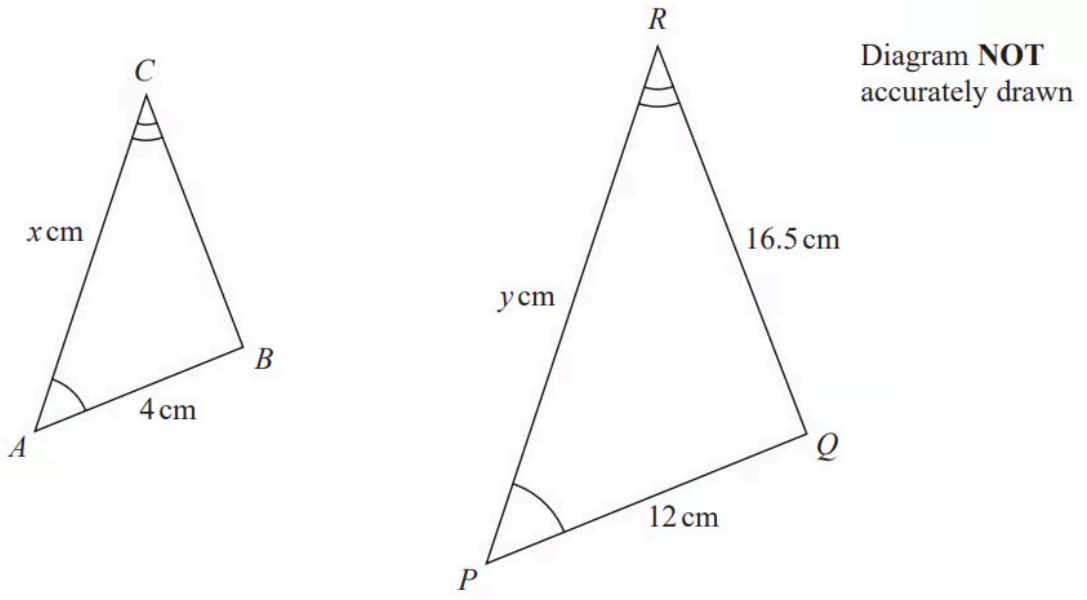


Diagram **NOT** accurately drawn

Triangle ABC is similar to triangle PQR.

$AB = 4$ cm $\quad PQ = 12$ cm $\quad RQ = 16.5$ cm $\quad AC = x$ cm $\quad PR = y$ cm

Calculate the length of BC.

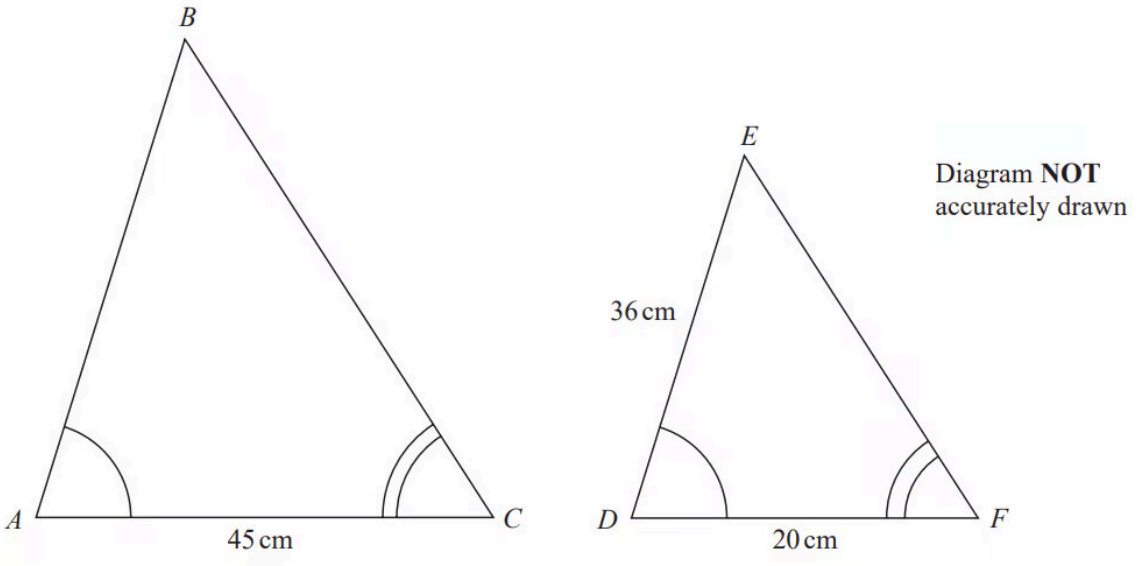
(2 marks)

(b) Write down an expression for y in terms of x

y = ...

(1 mark)

6 (a) ABC and DEF are similar triangles.



Work out the length of AB.

(2 marks)

(b) Given that $BC = 54$ cm,
work out the length of EF.

(2 marks)

7 The diagram shows two cylinders, **A** and **B**

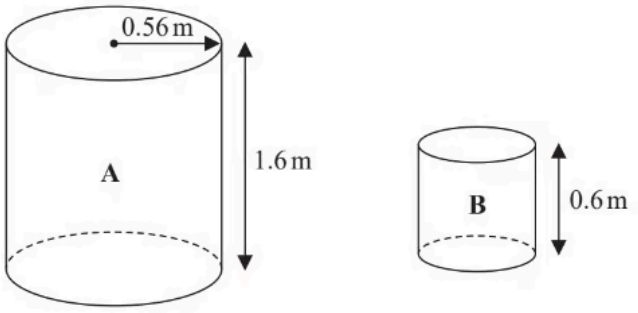


Diagram **NOT** accurately drawn

Cylinder **A** has height 1.6 m and radius 0.56 m.

Cylinder **B** is mathematically similar to cylinder **A**.
The height of cylinder **B** is 0.6 m.
Work out the radius of cylinder **B**.

..... m

(2 marks)

8 (a) ABC and DEF are similar triangles.

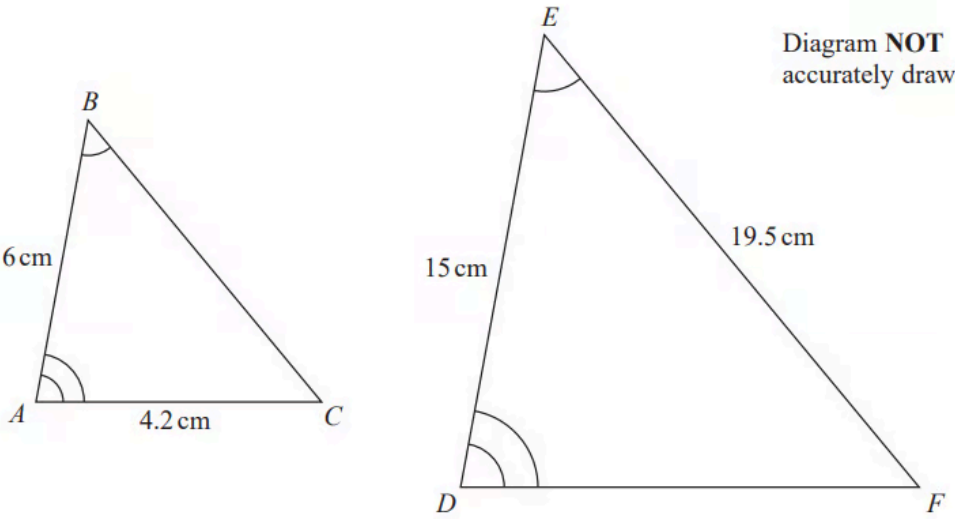


Diagram **NOT** accurately drawn

Work out the length of DF.

.....cm

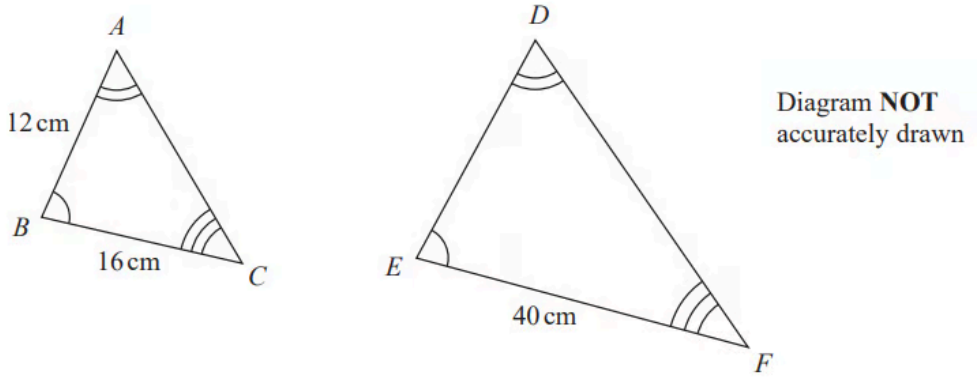
(2 marks)

(b) Work out the length of BC.

.....cm

(2 marks)

9 ABC and DEF are similar triangles.

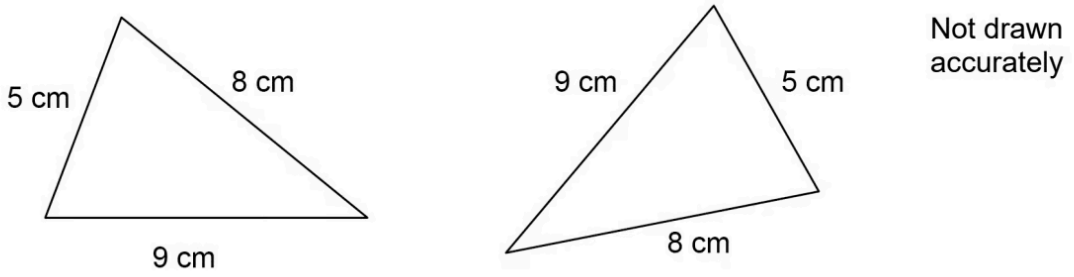


Work out the length of DE.

.....cm

(2 marks)

10



Choose the reason why these triangles are congruent.

A. ASA

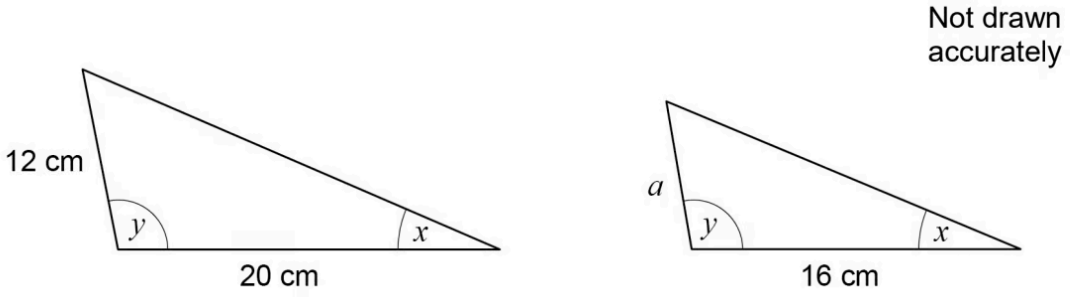
B. RHS

C. SAS

D. SSS

(1 mark)

11 These two triangles are similar.

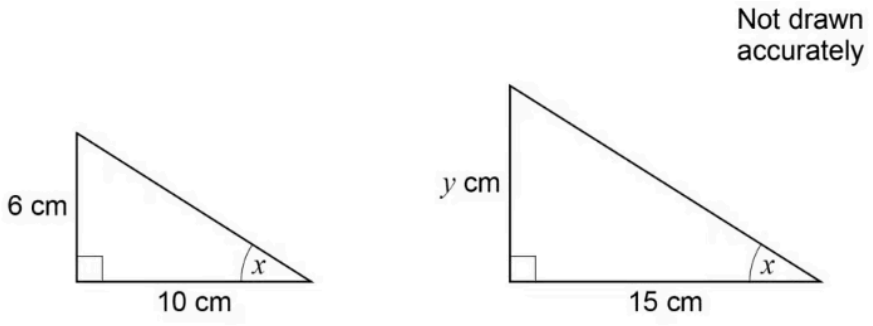


Work out the value of a.

.....cm

(2 marks)

12 Here are two right-angled triangles.



Choose the value of y.

A. 11

B. 7.5

C. 9

D. 4

(1 mark)

13 Which of these is not used to prove that triangles are congruent?

A. SSS

B. SAS

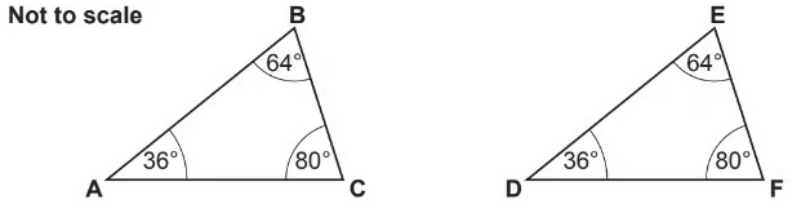
C. AAA

D. RHS

(1 mark)

14 Are these two triangles definitely congruent?

Give a reason.

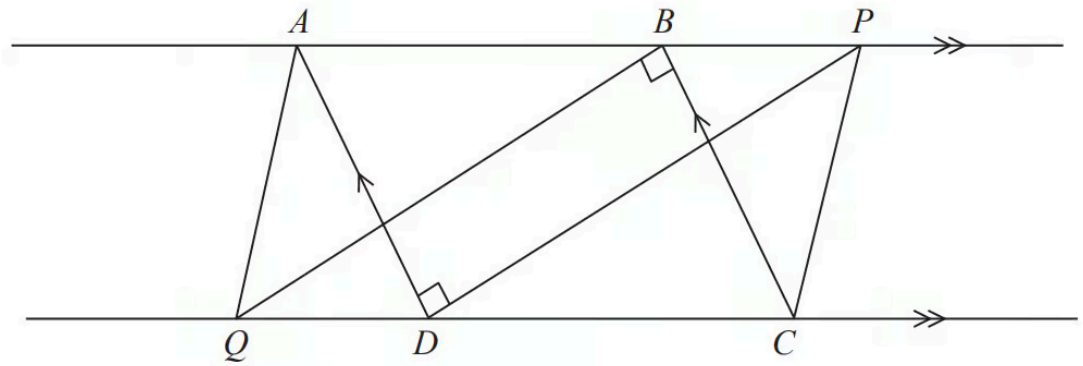


..... because .....

(1 mark)

Medium Questions

1 (a)



ABCD is a parallelogram.
ABP and *QDC* are straight lines.
Angle *ADP* = angle *CBQ* = 90^{o}.

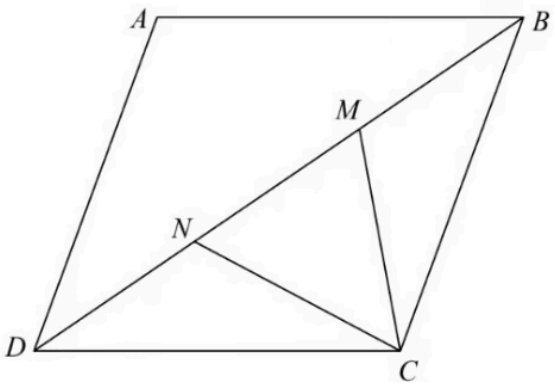
Prove that triangle *ADP* is congruent to triangle *CBQ*.

(3 marks)

(b) Explain why *AQ* is parallel to *PC*.

(2 marks)

2 $ABCD$ is a rhombus.

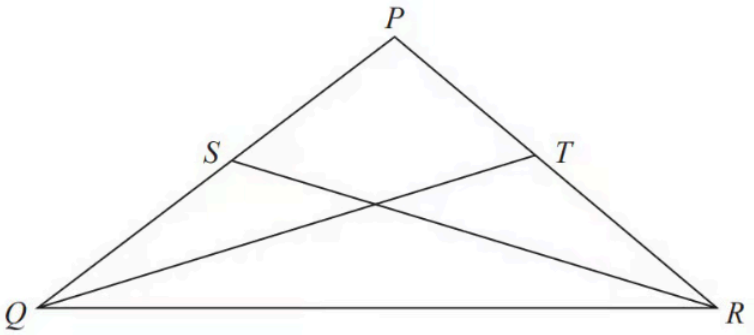


M and N are points on BD such that $DN = MB$.

Prove that triangle DNC is congruent to triangle BMC.

(3 marks)

3

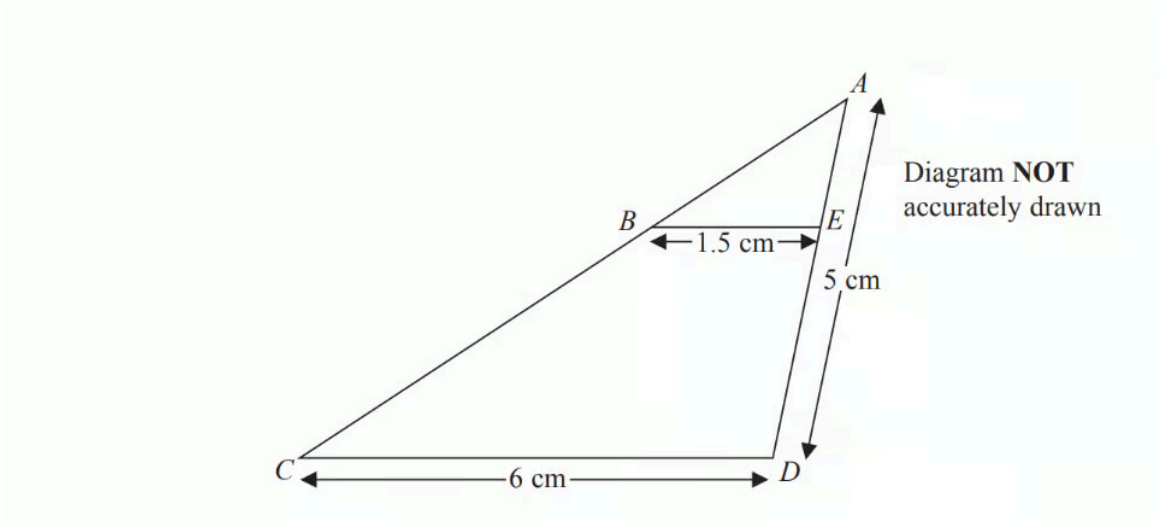


$PQ = PR$.
S is the midpoint of PQ.
T is the midpoint of PR.

Prove triangle QTR is congruent to triangle RSQ.

(3 marks)

4



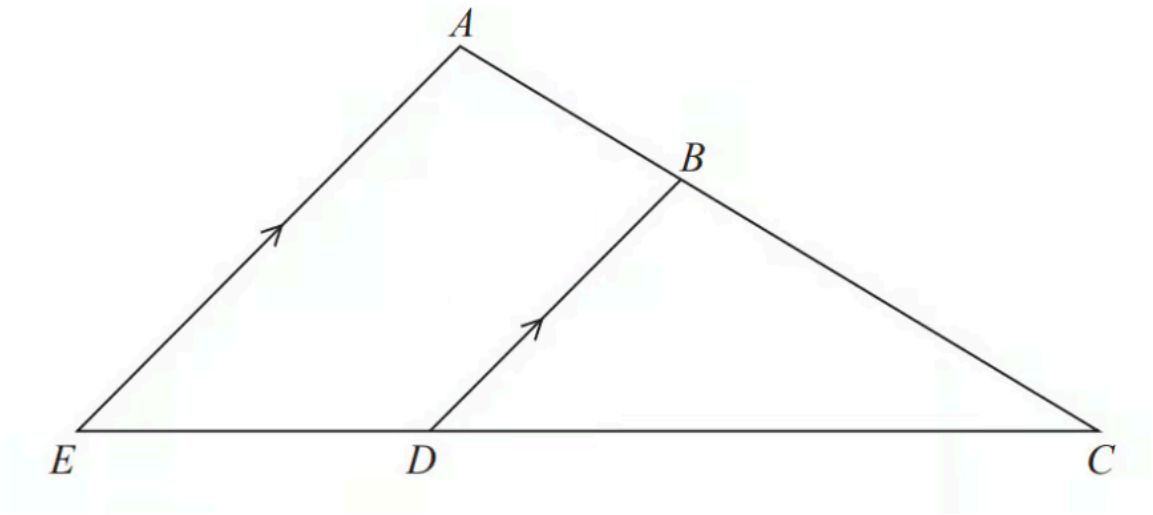
ABC and AED are straight lines.
BE and CD are parallel.

$BE = 1.5$ cm.
$CD = 6$ cm.
$AD = 5$ cm.

Calculate the length of ED.

(3 marks)

5 (a)



ABC and *EDC* are straight lines.
EA is parallel to *DB*.

EC = 8.1 cm.
DC = 5.4 cm.
DB = 2.6 cm,

Work out the length of *AE*.

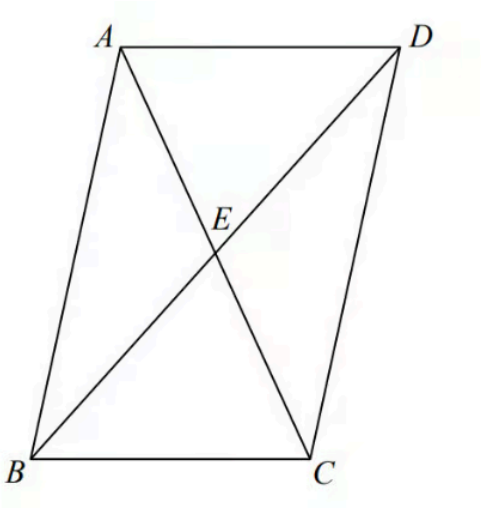
(2 marks)

(b) *AC* = 6.15 cm.

Work out the length of *AB*.

(2 marks)

6 $ABCD$ is a parallelogram.



E is the point where the diagonals AC and BD meet.

Prove that triangle ABE is congruent to triangle CDE.

(3 marks)

7 The diagram shows two water towers in Kuwait.

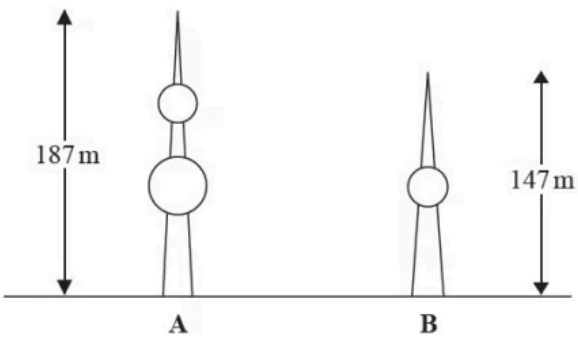


Diagram **NOT** accurately drawn

The real height of tower **A** is 187m.
The real height of tower **B** is 147m.

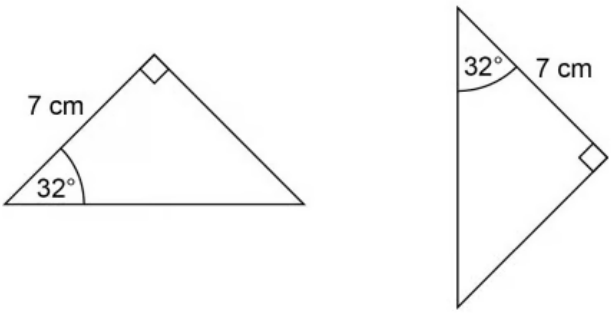
Ahmed makes a scale model of both towers.

The height of tower $\mathbf{A}$ on the scale model is 90cm.

Work out the height of tower $\mathbf{B}$ on the scale model.
Give your answer correct to the nearest centimetre.

(3 marks)

8



Not drawn accurately

Choose the reason why these triangles are congruent.

A. SSS

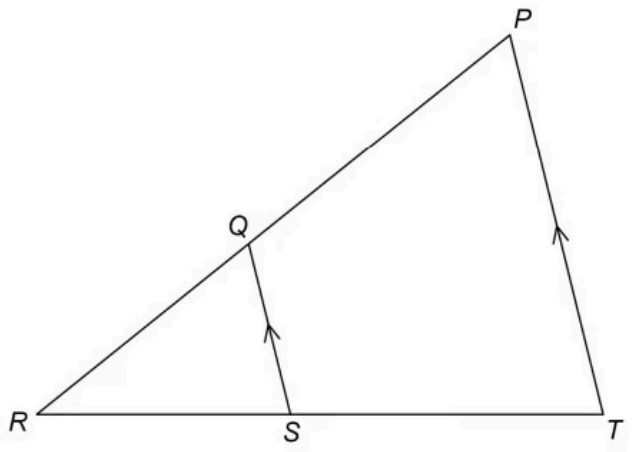
B. SAS

C. ASA

D. RHS

(1 mark)

9 PRT and QRS are similar triangles.



Not drawn accurately

Which of these is equivalent to $\frac{QR}{PR}$?

A. $\frac{RS}{ST}$

B. $\frac{QS}{PT}$

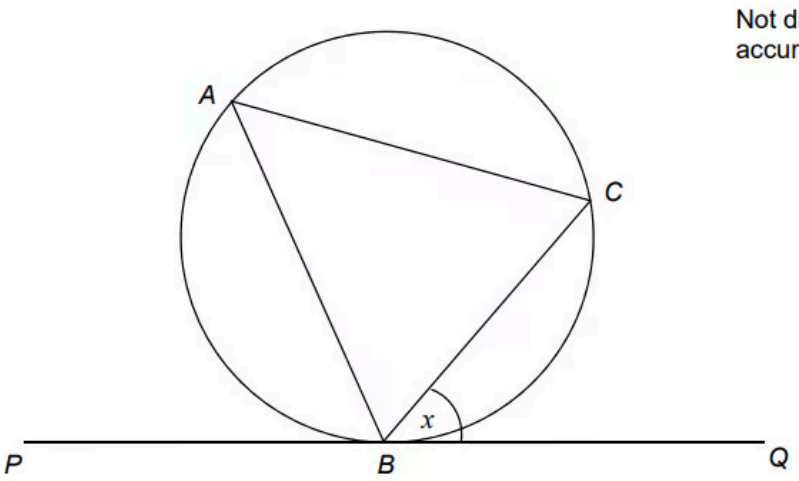
C. $\frac{PT}{QS}$

D. $\frac{RT}{RS}$

(1 mark)

10 A, B and C are points on a circle.

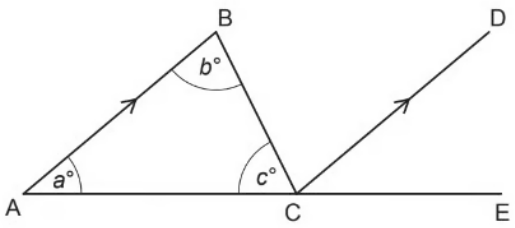
- BC bisects angle ABQ.
- PBQ is a tangent to the circle.



Angle $CBQ = x$
Prove that $AC = BC$

(3 marks)

11 (a) The diagram shows triangle ABC.
CD is parallel to AB. A, C and E lie in a straight line.
Angles of size $a°$, $b°$ and $c°$ are shown.



Not to scale

Insert $a°$, $b°$° or $c°$ to make this statement true.
Give a reason for your answer.

Angle DCE = ... because .....

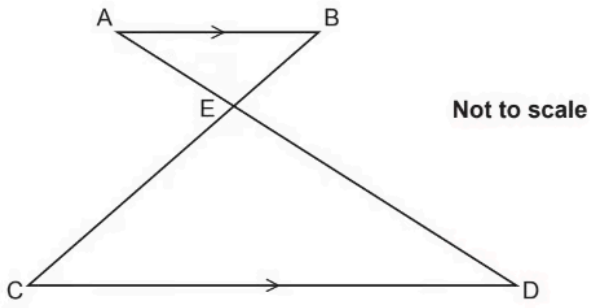
(2 marks)

(b) Use the diagram and the answer to part (a) to show that the angles of a triangle add up to 180°.

Give a reason for each statement you make.

(3 marks)

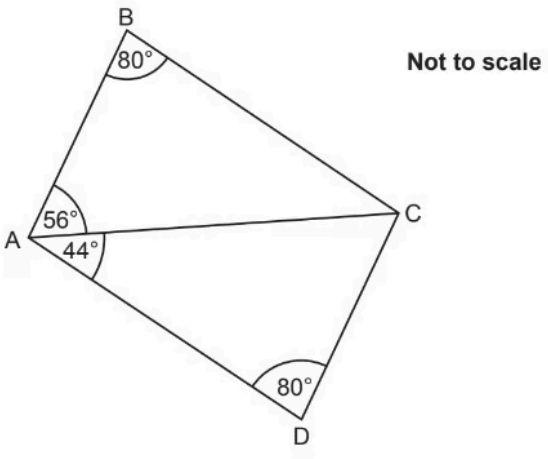
12 In the diagram AB is parallel to CD.
AED and BEC are straight lines.



Prove that triangle ABE is similar to triangle CDE.

(3 marks)

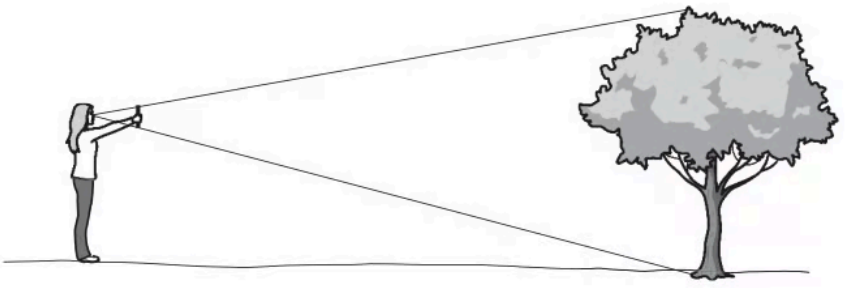
13 The diagram below shows two triangles.



Prove that triangle ABC is congruent to triangle ACD.

(4 marks)

14 (a) Anna estimates the height of a tree.



Anna holds a ruler vertically so the height of the tree is exactly covered by the ruler.
She is 20 metres from the tree.
The ruler is 30cm long.
The horizontal distance from her eyes to the ruler is 60 cm.

Calculate an estimate of the height of the tree.

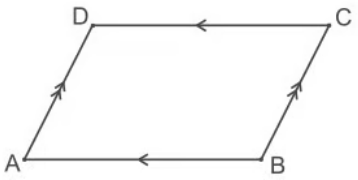
..... m

(3 marks)

(b) Give two reasons why this method may not be suitable to estimate the height of a very tall building.

(2 marks)

15 ABCD is a parallelogram.



Prove that triangle ABD is congruent to triangle CDB.

(3 marks)

Hard Questions

1

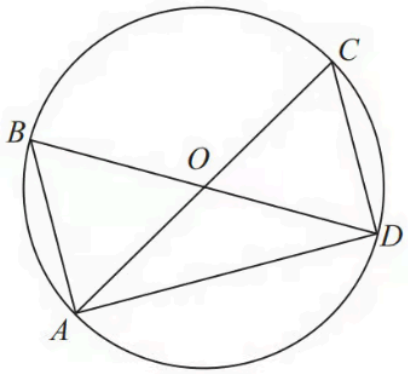


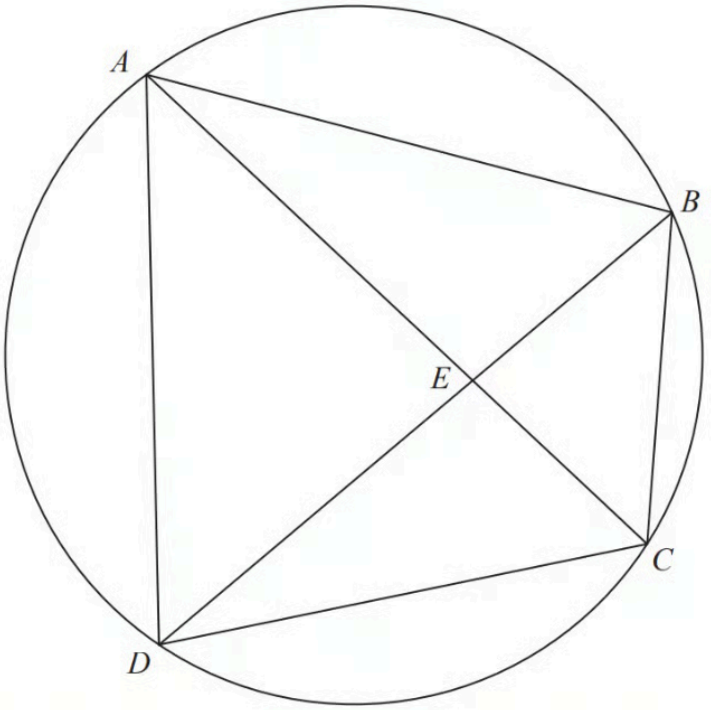
Diagram **NOT** accurately drawn

AOC and BOD are diameters of a circle, centre O.

Prove that triangle ABD and triangle DCA are congruent.

(3 marks)

2 A, B, C and D are four points on the circumference of a circle.



AEC and BED are straight lines. Prove that triangle ABE and triangle DCE are similar.
You must give reasons for each stage of your working.

(3 marks)

3 Steve has a photo and a rectangular piece of card.

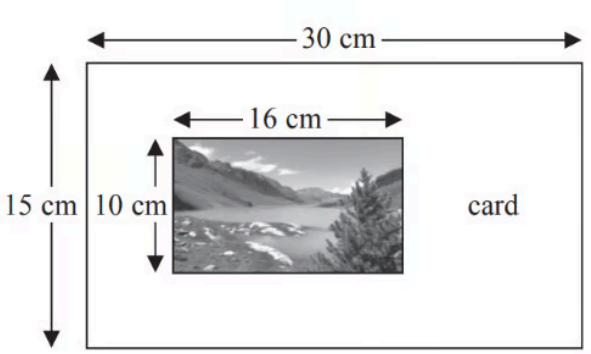
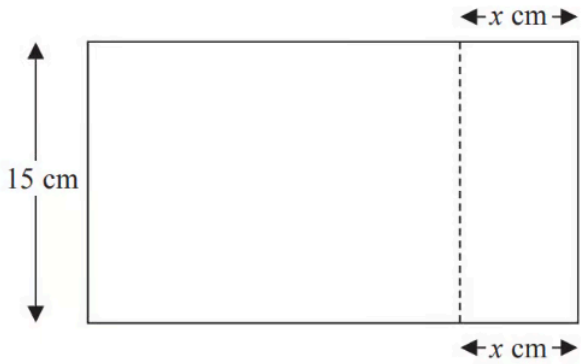


Diagram **NOT** accurately drawn

The photo is 16 cm by 10 cm.
The card is 30 cm by 15 cm.

Steve cuts the card along the dotted line shown in the diagram below.

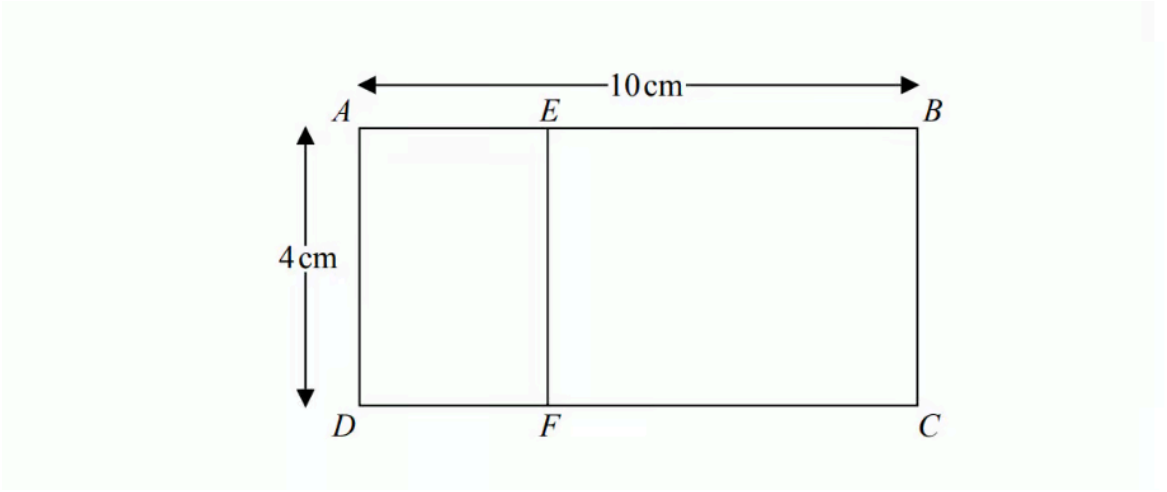


Steve throws away the piece of card that is 15 cm by x cm.
The piece of card he has left is mathematically similar to the photo.

Work out the value of x.

(3 marks)

4 Rectangle $ABCD$ is mathematically similar to rectangle $DAEF$.



$AB = 10$ cm.
$AD = 4$ cm.

Work out the area of rectangle $DAEF$.

(3 marks)

5 (a)

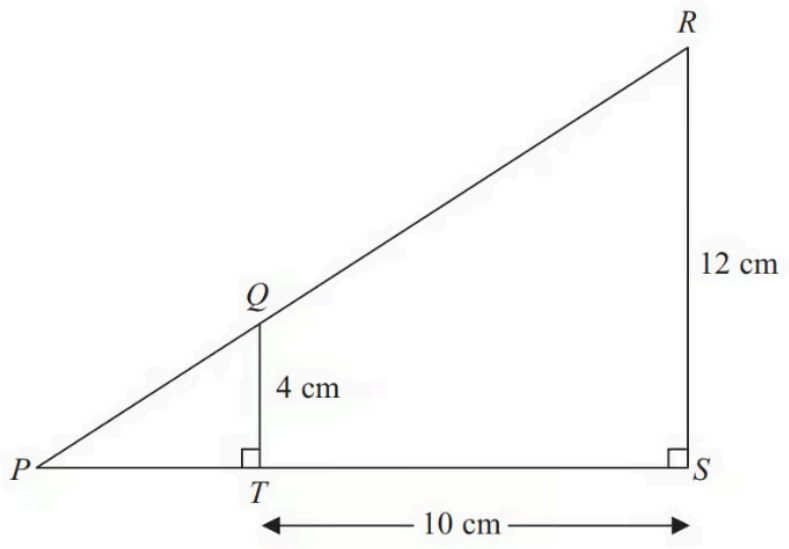


Diagram **NOT** accurately drawn

PQR and *PTS* are straight lines.
Angle *PTQ* = Angle *PSR* = 90^{o}.
QT = 4 cm
RS = 12 cm
TS = 10 cm

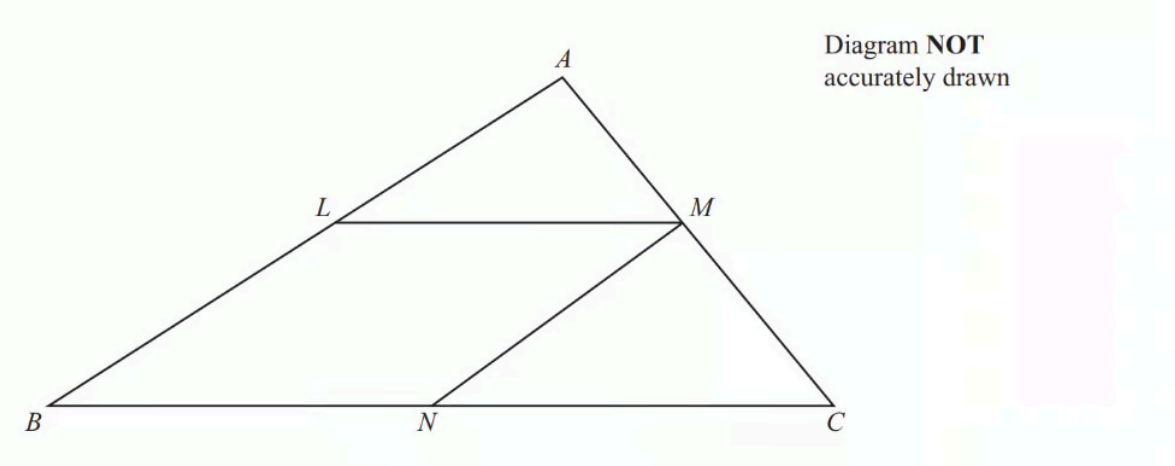
Work out the area of the trapezium *QRST*.

(2 marks)

(b) Work out the length of *PT*.

(3 marks)

6



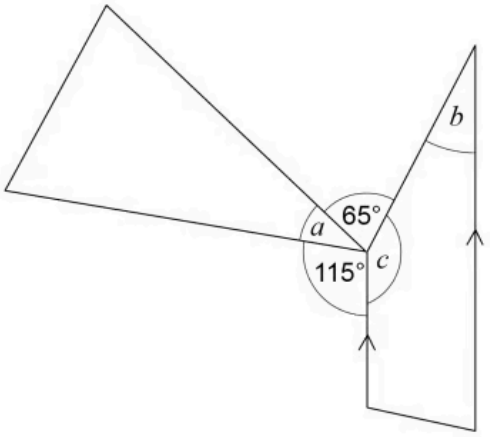
The diagram shows a triangle ABC.

$LMNB$ is a parallelogram where
L is the midpoint of AB,
M is the midpoint of AC,
and N is the midpoint of BC.

Prove that triangle ALM and triangle MNC are congruent.
You must give reasons for each stage of your proof.

(3 marks)

7 The diagram shows a triangle and a trapezium.

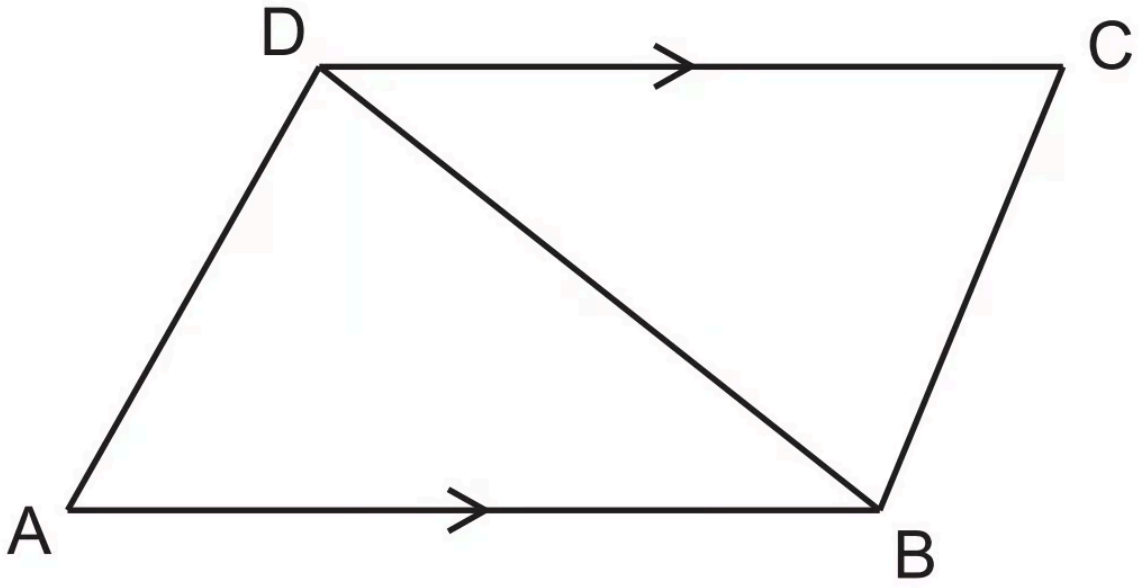


Not drawn accurately

Prove that $a = b$

(3 marks)

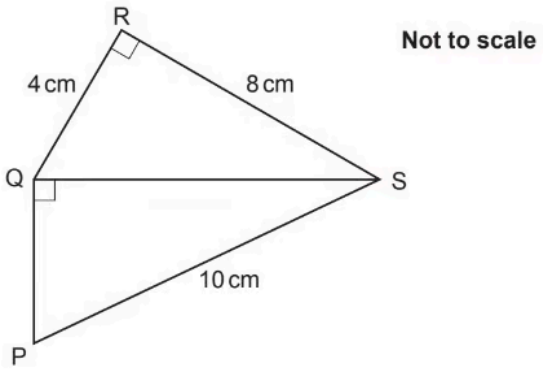
8 In the diagram, AB and DC are parallel lines of equal length.



Prove that angle DAB = angle BCD.

(4 marks)

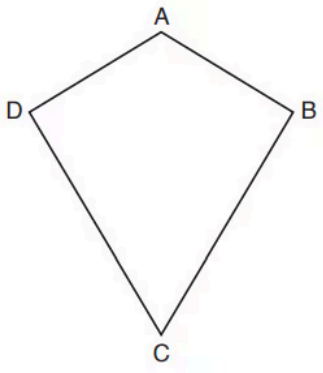
9 The diagram below shows two right-angled triangles.



Prove that triangles PQS and QRS are similar.

(5 marks)

10 ABCD is a quadrilateral.
AD = AB and CD = CB.



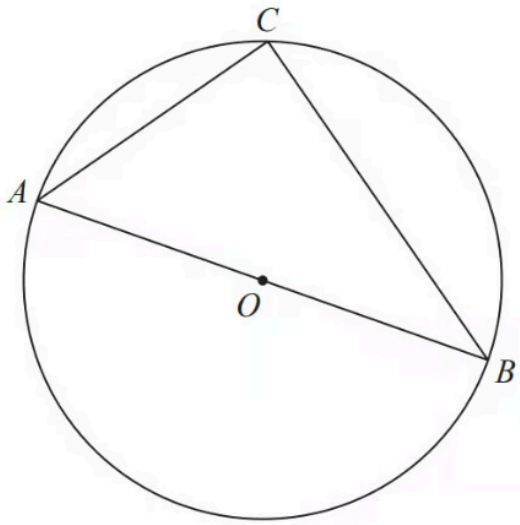
Not to scale

Prove that angle ADC is equal to angle ABC.

(4 marks)

Very Hard Questions

1

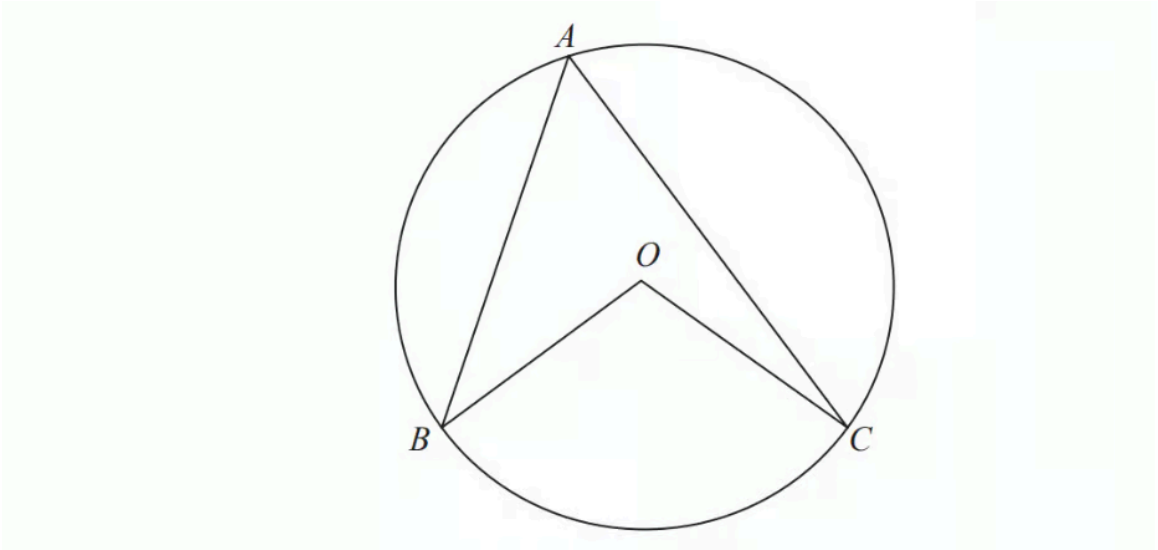


A, B and C are points on the circumference of a circle, centre O. AOB is a diameter of the circle.

Prove that angle ACB is 90°
You must **not** use any circle theorems in your proof.

(4 marks)

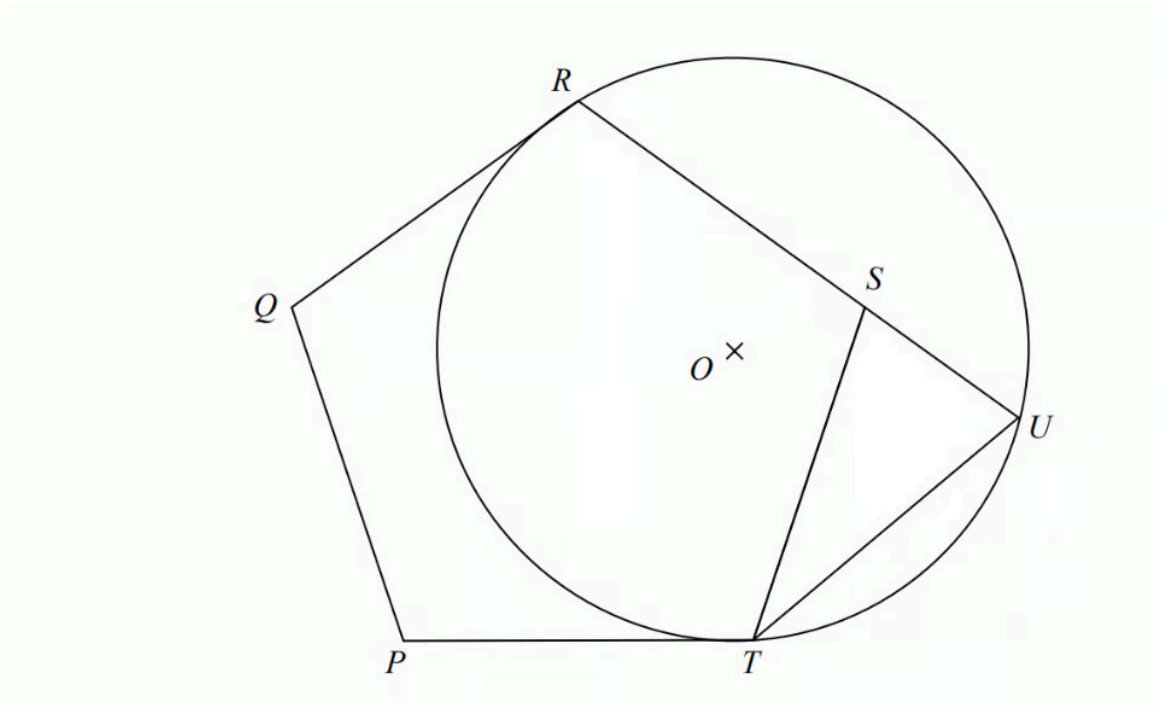
2 A, B and C are points on the circumference of a circle centre O.



Prove that angle BOC is twice the size of angle BAC.

(4 marks)

3

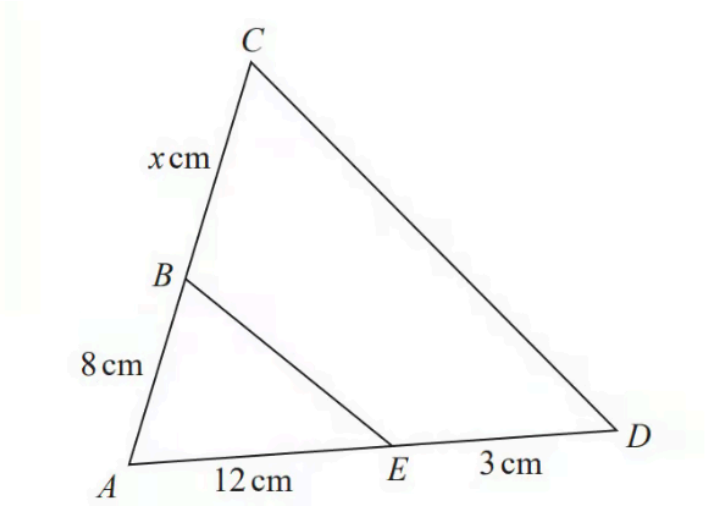


$PQRST$ is a regular pentagon.
R, U and T are points on a circle, centre O.
QR and PT are tangents to the circle.
RSU is a straight line.

Prove that $ST = UT$.

(5 marks)

4 The two triangles in the diagram are similar.



There are two possible values of x.

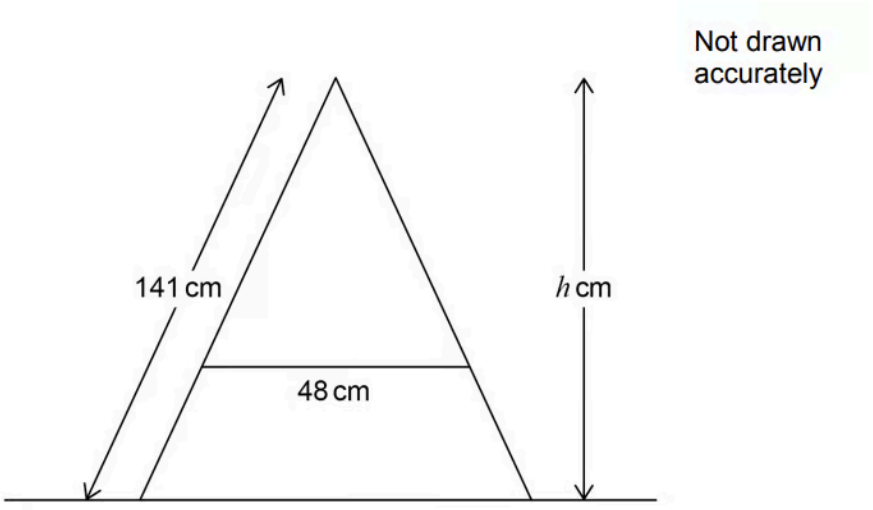
Work out each of these values.
State any assumptions you make in your working.

(5 marks)

5 The diagram shows the side view of a step ladder with a horizontal strut of length 48 cm.

The strut is one third of the way up the ladder.

The symmetrical cross section of the ladder shows two similar triangles.



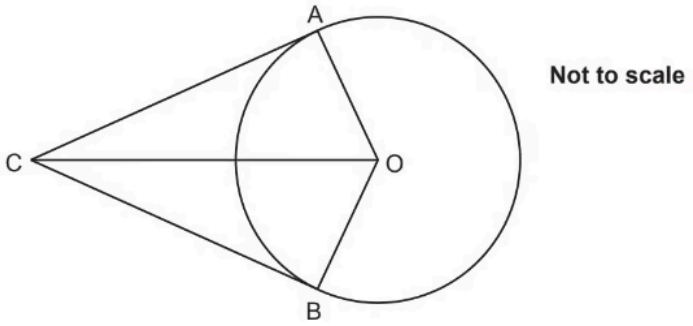
Work out the vertical height, h cm, of the ladder.

.....cm

(5 marks)

6 A and B are points on the circumference of a circle, centre O.

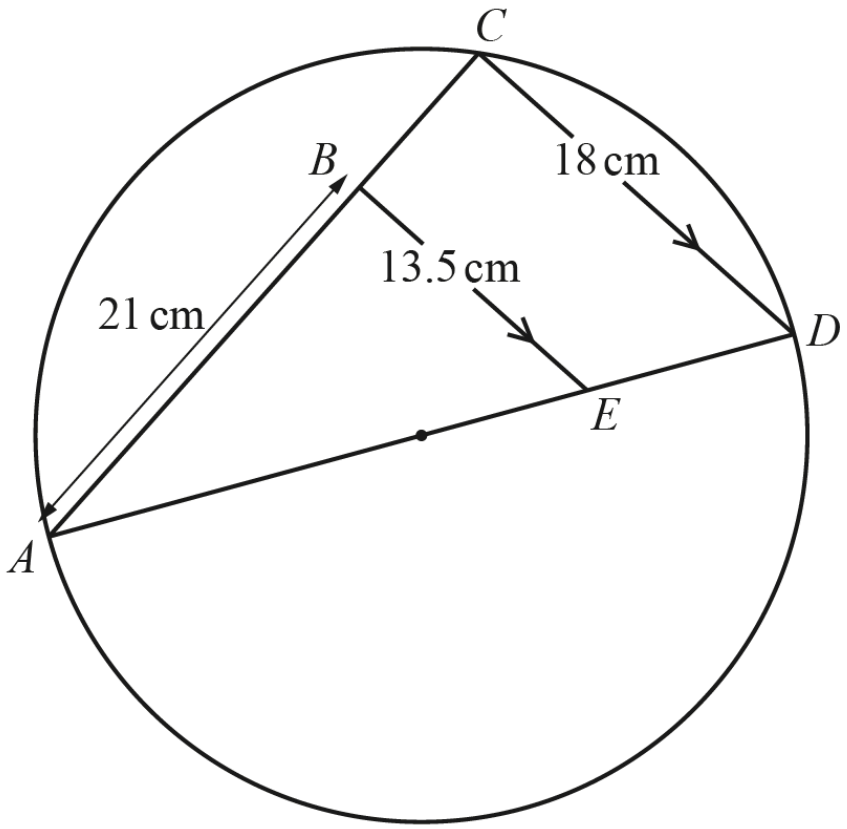
CA and CB are tangents to the circle.



Prove that triangle OAC is congruent to triangle OBC.

(4 marks)

7



NOT TO SCALE

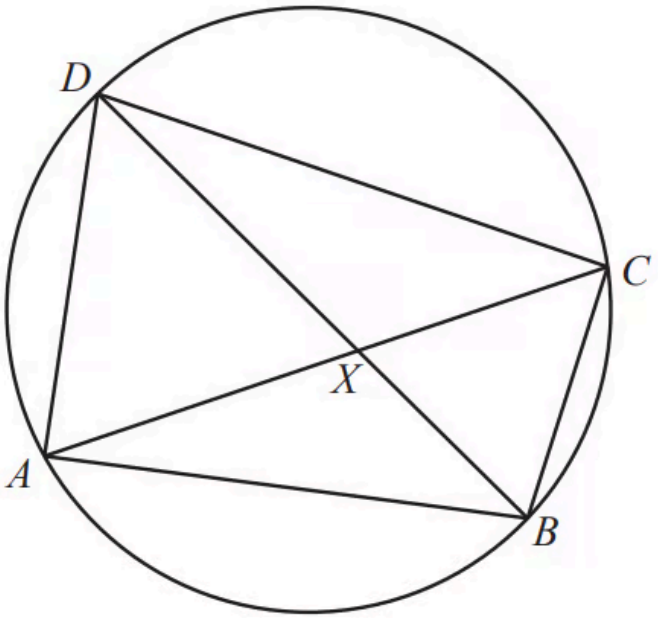
C lies on a circle with diameter AD.
B lies on AC and E lies on AD such that BE is parallel to CD.
AB = 21 cm, CD = 18 cm and BE = 13.5 cm.

Work out the radius of the circle.

..... cm

(5 marks)

8 (a)



NOT TO SCALE

The diagonals of the cyclic quadrilateral *ABCD* intersect at *X*.

Explain why triangle *ADX* is similar to triangle *BCX*.
Give a reason for each statement you make.

(3 marks)

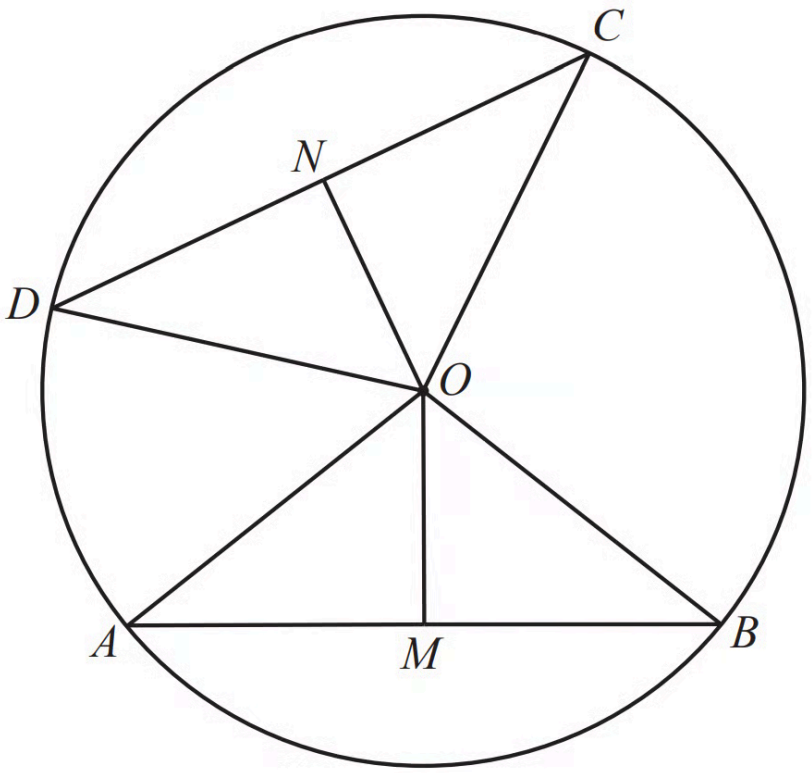
(b) *AD* = 10 cm, *BC* = 8 cm, *BX* = 5 cm, *CX* = 7 cm.

Calculate *DX*.

DX = ... cm

(2 marks)

9



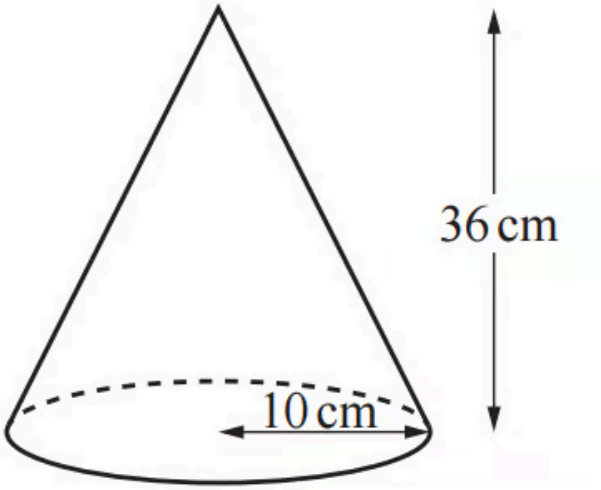
NOT TO SCALE

A, *B*, *C* and *D* are points on the circle, centre *O*.
M is the midpoint of *AB* and *N* is the midpoint of *CD*.
OM = *ON*

Explain, giving reasons, why triangle *OAB* is congruent to triangle *OCD*.

(3 marks)

10 (a)



NOT TO SCALE

A solid metal cone has radius 10 cm and height 36 cm.

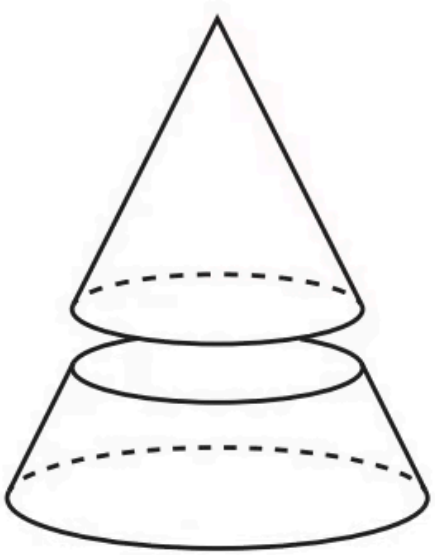
Calculate the volume of this cone.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

..... cm^3

(2 marks)

(b) The cone is cut, parallel to its base, to give a smaller cone.



NOT TO SCALE

The volume of the smaller cone is half the volume of the original cone.

The smaller cone is melted down to make two different spheres.

The ratio of the radii of these two spheres is 1 : 2.

Calculate the radius of the smaller sphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

..... cm

(4 marks)